

Annotated Examples 7

Leonard 2009

This is the example used in Leonard 2009 in char 2 to show that the P-module generating set need not be a P-module basis, given that there is a non-trivial P-linear dependence among the generators. (In the char 0 version there are two such.) I_6 in Normalization of rings is a poorly stated version of this.

Consider this using normal in SINGULAR 3-1-0: [Note that the denominator is in terms of the most dependent variable, z, rather than in terms of the independent variables, x2,x1; and that the T(i)'s should probably be reversed in order in general, given that the earlier they are generated (higher i), the less likely they are to be necessary, given that they rate to have a smaller denominator. Here T(6),T(5),T(4) should be considered unnecessary.]

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intmat A[5][5]=1,1,1,0,0,1,1,0,0,0,1,0,0,0,0,19,15,12,9,9,12,9,9,9,0;
> ring r=2,(z,y1,y2,x2,x1),M(A);
> ideal i=y1^2+y2*x2*x1,
      y2*y1+y1+x2^2*x1+x2*x1^2,
      y2^2+y2+y1*(x2+x1),
      z^3+(y2+y1)*(x2+1)*x1*z+(y2*(x2*x1+1)+y1*(x2+x1))*x2^2*x1;
> list nor=normal(i);
> nor;
[1]:
[1]:
// characteristic : 2
// number of vars : 12
//   block   1 : ordering dp
//           : names   T(1) T(2) T(3) T(4) T(5) T(6) T(7)
//   block   2 : ordering M
//           : names   z y1 y2 x2 x1
//           : weights  1  1  1  0  0
//           : weights  1  1  0  0  0
//           : weights  1  0  0  0  0
//           : weights 19 15 12  9  9
//           : weights 12  9  9  9  0
//   block   3 : ordering C
[2]:
[1]:
_[1]=z*y1^2*y2+z*y1^2*x2+z*y1^2+z*y1*x2*x1^2
_[2]=z*y1^2*x2+z*y1*y2*x2^2+z*y1*x2^3+z*y1*x2^2
_[3]=z*y1^2*x2^2+z*y1^2*x2+z*y1*x2^3+z*y1*x2^2
_[4]=z*y1^2*y2*x2+z*y1^2*x2*x1+z*y1*x2^3+z*y1*x2^2
_[5]=z*y1^3*x2+z*y1^2*x2*x1+z*y1*x2^3+z*y1*x2^2
_[6]=z*y1^4+z*y1^3*y2+z*y1^3+z*y1^2*x2*x1
_[7]=z*y1^2*x2+z*y1^2+y1^3*x2^3+y1^3*x2+z*y1*y2+z*y1*x2^2+z*y1*x1+z*y1+y1^2*x2^4+y1^2
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_ [8]=z^2

```
> def R=nor[1][1];
> setring R;
> normap;
normap[1]=z
normap[2]=y1
normap[3]=y2
normap[4]=x2
normap[5]=x1
> norid;
norid[1]=T(1)*x1+T(1)+T(2)*x1+T(4)+T(5)
norid[2]=T(3)*x1+T(4)*x1+T(6)
norid[3]=T(1)*x2+T(1)*x1+T(3)+T(4)
norid[4]=T(1)*y2+T(4)+T(5)
norid[5]=T(1)*x1+T(1)+T(2)*y2+T(2)*x2+T(3)*x2+T(3)*x1
norid[6]=T(1)*x1+T(1)+T(2)*x2+T(2)+T(3)*y2+T(3)*x1+T(3)+T(4)*x2
norid[7]=T(1)*x1+T(1)+T(2)*x2+T(2)+T(3)+T(4)*y2+T(5)*x2+T(5)*x1+T(6)
norid[8]=T(1)*y1+T(1)*x1+T(1)+T(3)*x1+T(4)+T(5)
norid[9]=T(1)*x1+T(1)+T(2)*y1+T(3)*x2+T(3)*x1+T(4)*x2+T(4)+T(5)
norid[10]=T(1)*x1+T(1)+T(3)*y1+T(3)*x2+T(3)*x1+T(4)+T(5)*x2+T(5)
norid[11]=T(2)*x2+T(2)+T(3)*x2+T(3)*x1+T(3)+T(4)*y1+T(5)*y2+T(5)*x1+T(6)
norid[12]=T(1)*x1+T(1)+T(2)*x2+T(2)+T(3)*x2+T(3)+T(4)+T(5)*y1+T(5)*y2+T(5)*x1+T(5)+T(6)*x2+
norid[13]=T(1)*x1+T(1)+T(2)*x2+T(2)+T(3)*x2*x1+T(3)*x1^2+T(3)*x1+T(4)*x2+T(5)*y2+T(5)+T(6)*
norid[14]=T(1)*x1+T(1)+T(2)*x2+T(2)+T(3)*x1+T(4)*x2+T(5)*y2+T(5)*x2*x1+T(5)*x1^2+T(5)+T(6)*
norid[15]=T(2)*x2+T(2)+T(3)*x1+T(3)+T(5)*y2*x1+T(5)*y2+T(5)*x2+T(5)*x1+T(6)*y1+T(6)*y2+T(6)*
norid[16]=T(1)*x1+T(1)+T(3)*x1+z^2+y1^2+y1*x2*x1+y1*x1+y2*x1
norid[17]=T(1)*x1^2+T(1)*x1+T(2)*z+T(2)*x2^3+T(2)*x2^2+T(2)*x2+T(2)+T(3)*x1^3+T(3)*x2^2+T(3)*
norid[18]=T(1)*z^2+T(1)*z*x1^2+T(1)*z*x1+T(1)*x1^3+T(1)*x1^2+T(2)*z^2+T(2)*z*x2+T(2)*z+T(2)*
norid[19]=T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(1)+T(2)*x2^2+T(2)*x2+T(3)*z+T(3)*x2^3+T(3)*x1^3+T(3)*
norid[20]=T(1)*x1^2+T(1)*x1+T(2)*x2+T(2)+T(3)*x2^2+T(3)*x1^2+T(3)*x2+T(3)+T(5)*y2+T(5)*x2^2+
norid[21]=T(1)+T(3)*x1+T(4)+T(5)+T(7)*x1+z*y1*x2+z*y2+z*x2*x1+z
norid[22]=T(1)*z+T(1)*x1^2+T(1)*x1+T(2)*x2+T(2)+T(3)*x1^2+T(3)+T(4)*x2+T(4)+T(5)*y2+T(5)+z^
norid[23]=T(1)*z^2*x1+T(1)*z^2+T(1)*x1+T(1)+T(2)*z^2*x2+T(2)*z^2+T(2)*z+T(3)*x1^3+T(4)*z^2+
norid[24]=T(1)*z+T(2)*x2+T(2)+T(3)*x2^2+T(3)*x2+T(3)+T(5)*y2+T(5)*x2^2+T(7)*z+y1^2*y2+z^2*x
norid[25]=T(1)*z^2+T(1)*x1^2+T(1)*x1+T(2)*z^2+T(2)*x2^2+T(2)*x2+T(3)*x2+T(4)*x2+T(5)*y2*x2+
norid[26]=T(1)*z+T(1)*x1^3+T(1)+T(2)*z+T(2)*x2^2+T(2)+T(3)*z+T(3)*x1^3+T(3)*x2^2+T(3)*x1^2+
norid[27]=T(1)*x1^3+T(1)*x1+T(2)*z+T(2)*x2^2+T(2)+T(3)*z+T(3)*x1^3+T(3)*x1^2+T(3)*x2+T(3)+T
norid[28]=T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(1)+T(2)*x2^2+T(2)*x2+T(3)*z^2+T(3)*x1^2+T(3)*x2+T(3)
norid[29]=T(1)*z+T(1)*x1^2+T(1)+T(2)*z+T(2)*x2^2+T(2)*x2+T(3)*z+T(3)*x1^3+T(3)*x2^2+T(4)*z+
norid[30]=T(1)*z+T(1)*x1^4+T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(2)*z+T(2)*x2^2+T(2)*x2+T(3)*z*x1+T
norid[31]=T(1)*z^2+T(1)*z*x1^3+T(1)*z*x1+T(1)*x1^3+T(1)*x1^2+T(2)*z+T(2)*x2^2+T(2)+T(3)*z^2+
norid[32]=T(1)*z+T(1)*x1^3+T(1)*x1^2+T(2)*x2+T(2)+T(3)*z^2*x2+T(3)*z^2+T(3)*z*x1+T(3)*z+T(3)
norid[33]=T(1)*z*x1+T(1)*z+T(1)*x1^3+T(1)*x1^2+T(1)+T(2)*x2^2+T(2)*x2+T(3)*z^2+T(3)*z*x1+T(
norid[34]=T(2)*z^2+T(2)*z+T(3)*z^2*x1+T(3)*x1^3+T(3)*x1+T(4)*z^2*x2+T(4)*z+T(4)*x2^3+T(4)*x
norid[35]=T(1)*z^3+T(1)*z^2+T(1)*z*x1+T(1)*z+T(1)*x1^2+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T
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norid[36]= $T(1)*z*x1+T(1)*z+T(1)*x1^3+T(1)*x1^2+T(2)*z^2+T(2)*z+T(2)*x2^2+T(2)*x2+T(3)*z^2*x1$
 norid[37]= $T(1)*z^3+T(1)*z^2+T(1)*z+T(1)*x1^3+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*x2^2+T(2)*z^2*x1$
 norid[38]= $T(1)*z^2*x1+T(1)*z^2+T(1)*z+T(1)*x1^3+T(1)*x1+T(2)*z+T(2)*x2^2+T(2)*x2+T(3)*z^2*x1$
 norid[39]= $T(1)*z+T(1)*x1^2+T(1)*x1+T(2)*z^2+T(2)*z*x2^2+T(3)*z^3+T(3)*z^2*x2+T(3)*z^2*x1+T(3)*z^2*x1$
 norid[40]= $T(1)*z*x1^2+T(1)*z*x1+T(1)*x1^3+T(2)*z+T(2)*x2+T(2)+T(3)*z^2*x2+T(3)*z^2*x1+T(3)*z^2*x1$
 norid[41]= $T(1)*z+T(1)*x1^4+T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(2)*x2+T(2)+T(3)*z*x2+T(3)*x2^4+T(3)*z^2*x1$
 norid[42]= $T(1)*z^3+T(1)*z^2+T(1)*z*x1+T(1)*x1^2+T(1)*x1+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[43]= $T(1)*z^3+T(1)*z^2+T(1)*z*x1+T(1)*x1^2+T(1)*x1+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[44]= $T(1)*z^3+T(1)*z^2+T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[45]= $T(2)+T(3)*x2+T(3)+T(4)*x2+T(4)+T(5)+T(7)*y2+z*y1+z*y2*x2+z*x2^3+z*x2^2*x1$
 norid[46]= $T(1)*z^2+T(1)*z*x1+T(1)*z+T(1)*x1^2+T(1)*x1+T(2)*x2^2+T(2)+T(3)*z^2*x2+T(3)*z^2+T(3)*z^2*x1$
 norid[47]= $T(1)*x1+T(1)+T(3)*x1+T(4)*x2+T(4)+T(5)*x2+T(5)+T(7)*y1+z*y2^2+z*y1*x1+z*y2*x2^2+z*x2^2*x1$
 norid[48]= $T(1)+T(2)*z^2+T(2)*z+T(2)*x2+T(2)+T(3)*z^2*x2+T(3)*z*x2^3+T(3)*z*x2+T(3)*x1^3+T(3)*z^2*x1$
 norid[49]= $T(1)*z^3+T(1)*z^2+T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[50]= $T(1)*z^2*x1+T(1)*x1+T(1)+T(2)*x2+T(2)+T(3)*z^2*x2+T(3)*z^2+T(3)*z*x2^3+T(3)*z*x2+T(3)*z^2*x1$
 norid[51]= $T(1)*z^3+T(1)*z^2*x1+T(1)*z^2+T(1)*z*x1+T(1)*x1^2+T(1)*x1+T(1)+T(2)*z^3+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[52]= $T(1)*z^3+T(1)*x1+T(1)+T(2)*z^3+T(2)*z*x2^2+T(3)*z^2*x1^2+T(3)*z^2+T(3)*z*x2^2+T(3)*z^2*x1$
 norid[53]= $T(1)*z^2*x1^2+T(1)*z^2+T(1)*z*x1^2+T(1)*x1^3+T(1)*x1^2+T(1)*x1+T(2)*z^2+T(2)*z^2*x1$
 norid[54]= $T(1)*z*x1^2+T(1)*z*x1+T(1)*z+T(1)*x1^3+T(1)*x1+T(2)*z+T(2)*x2^2+T(2)+T(3)*z^3*x2+T(3)*z^2*x1$
 norid[55]= $T(1)*z^3+T(1)*z^2*x1+T(1)*z^2+T(1)*z+T(1)*x1^3+T(1)*x1+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[56]= $T(1)*z^2*x1+T(1)*z^2+T(1)*z*x1^2+T(1)*z*x1+T(1)*x1^2+T(1)*x1+T(1)*x1+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2$
 norid[57]= $T(1)*z^3+T(1)*z^2*x1+T(1)*z*x1^2+T(1)*z+T(1)*x1^4+T(1)*x1^3+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[58]= $T(1)*z^4+T(1)*z^2*x1^2+T(1)*z^2+T(1)*z*x1^2+T(1)*x1^2+T(1)+T(2)*z^4+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[59]= $T(1)*z^2+T(1)*z*x1^3+T(1)*z*x1+T(1)*x1^5+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[60]= $T(1)*z*x1^2+T(1)*z*x1+T(1)*x1^2+T(1)+T(2)*z^2+T(2)*z*x2^2+T(2)*z*x2+T(2)*x2^2+T(2)*z^2+T(2)*z*x1$
 norid[61]= $T(1)*z^3+T(4)*z^3+T(5)*z^3+T(6)*y1^2*y2*x2+T(6)*y1^2*y2*x1+T(6)*z*y1*x2+T(6)*z*y1$
 norid[62]= $T(1)*z^2+T(1)*z*x1+T(1)*x1^3+T(1)*x1^2+T(1)+T(2)*z^2+T(3)*z^2*x2+T(3)*z^2*x1+T(3)*z^2*x1$
 norid[63]= $T(1)*z^3+T(1)*z^2+T(1)*z*x1^2+T(1)*z*x1+T(1)*x1^2+T(1)*x1+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[64]= $T(1)*z^2*x1+T(1)*z*x1+T(1)*x1^3+T(2)*z^2+T(2)*z+T(2)*x2^2+T(2)*x2+T(3)*z^2+T(3)*z*x2^2+T(3)*z^2*x1$
 norid[65]= $T(1)*z^3+T(1)*z^2*x1^2+T(1)*x1+T(1)+T(2)*z^3+T(2)*z*x2^2+T(2)*z+T(3)*z^2*x2+T(3)*z^2*x1$
 norid[66]= $T(1)*z^3+T(1)*z*x1^2+T(1)*z*x1+T(1)*z+T(1)*x1^2+T(1)*x1+T(1)+T(2)*z^3+T(2)*z^2+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[67]= $T(1)*z^4+T(1)*z^3*x1+T(1)*z^2*x1^2+T(1)*z^2+T(1)*z*x1^2+T(1)*z*x1+T(1)*z+T(1)*x1+T(1)*x1+T(2)*z^4+T(2)*z^3*x1$
 norid[68]= $T(1)*z^3*x1^2+T(1)*z^3*x1+T(1)*z^2*x1^2+T(1)*z+T(1)*x1^3+T(2)*z^3+T(2)*z*x2^2+T(2)*z^2*x1+T(2)*z^2*x1$
 norid[69]= $T(1)*z^3*x1^2+T(1)*z^3*x1+T(1)*z^2*x1^2+T(1)*z+T(1)*x1^2+T(1)+T(2)*z^2+T(2)*z+T(2)*z*x2^2+T(2)*z^2*x1$
 norid[70]= $T(1)*z^3*x1+T(1)*z^3+T(1)*z*x1^2+T(1)*z*x1+T(1)*z+T(1)*x1+T(1)+T(2)*z+T(3)*z^3*x2+T(3)*z^2*x1$
 norid[71]= $T(1)^2+T(1)*y1+T(1)*x1^2+T(1)*x1+T(4)+T(5)+T(6)+z*y1*y2+z^2+z*y1+y1^2+z*x2*x1^2+y1^2+z*x2*x1^2+y1^2$
 norid[72]= $T(1)*T(2)+T(1)*y1+T(1)*x1^2+T(1)+T(2)*y1+T(3)*y1+T(3)+T(4)+T(6)+z*y1*y2+z^2+z*y1+y1^2+z*x2*x1^2+y1^2$
 norid[73]= $T(2)^2+T(1)+T(2)*y1+T(3)*x2+T(3)+T(4)+T(7)*x2^2+z*y1*y2+y1^2*y2+z^2*x2+z*y1+y1^2+z*x2*x1^2+y1^2$
 norid[74]= $T(1)*T(3)+T(1)*y1+T(1)*x1^2+T(1)+T(3)+T(5)+T(6)+z*y1*y2+y1^2*y2+z^2*x2+z*y1*x2^2+y1^2+z*x2*x1^2+y1^2$
 norid[75]= $T(2)*T(3)+T(1)+T(2)*y1*x2+T(2)*y1+T(2)+T(3)*y1+T(3)*x2+T(3)+T(4)+T(7)*x2^2+z*y1*y2+y1^2+z*x2*x1^2+y1^2$
 norid[76]= $T(3)^2+T(1)+T(2)*y1*x2+T(2)*y1+T(2)+T(3)*y1*x2+T(3)*x2+T(3)+T(4)+T(7)*x2^2+z*y1*y2+y1^2+z*x2*x1^2+y1^2$
 norid[77]= $T(1)*T(4)+T(1)*x1^2+T(1)*x1+T(2)*y1+T(3)*y1+T(3)+T(4)+T(5)*x1+T(6)+y1^3+z^2*y2+z*y1+y1^2+z*x2*x1^2+y1^2$
 norid[78]= $T(2)*T(4)+T(1)*y1+T(1)*x1+T(2)*y1+T(2)+T(3)*y1*x2+T(3)*x2+T(4)+T(5)*x1+T(5)+T(7)*y1+y1^2+z*x2*x1^2+y1^2$
 norid[79]= $T(3)*T(4)+T(1)*y1+T(1)*x1+T(2)*y1*x2+T(2)*y1+T(2)+T(3)*y1*x2+T(3)*y1+T(3)*x2+T(5)*y1+y1^2+z*x2*x1^2+y1^2$
 norid[80]= $T(4)^2+T(1)*y1+T(1)+T(2)*y1+T(2)+T(3)*y1+T(3)*x2+T(3)+T(5)*y1+T(5)+T(6)*y2+T(6)*x2+T(6)*y1+y1^2+z*x2*x1^2+y1^2$
 norid[81]= $T(1)*T(5)+T(1)*x1^2+T(1)*x1+T(3)+T(5)*x1+T(5)+T(6)+z^2*y1+z*y1^2*x2+y1^3+z*y1*y2*x1^2+y1^2+z*x2*x1^2+y1^2$

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norid[82]=T(2)*T(5)+T(1)*y1+T(1)*x1+T(2)*y1*x2+T(2)+T(3)*y1+T(3)*x2+T(4)+T(5)*x1+T(5)+T(7)*
norid[83]=T(3)*T(5)+T(1)*y1+T(1)*x1+T(2)*y1*x2+T(2)*y1+T(2)+T(3)*y1*x2+T(3)*y1+T(3)*x2+T(5)*
norid[84]=T(4)*T(5)+T(1)*y1+T(1)+T(2)*y1*x2+T(2)*y1+T(2)+T(3)*y1*x2+T(3)*y1+T(3)*x2+T(3)+T(4)*
norid[85]=T(5)^2+T(1)*y1+T(1)+T(2)*y1*x2+T(2)+T(3)*y1*x2+T(3)*x2+T(3)+T(5)*y1+T(5)+T(6)*y2+
norid[86]=T(1)*T(6)+T(1)*y1+T(3)*y1+T(5)*y1+T(6)*y2+T(6)*x1+z*y1^3+y1^3*x1+y1^3+z^2*y2*x1+z
norid[87]=T(2)*T(6)+T(1)*x1+T(1)+T(2)*y1*x2+T(2)*y1+T(3)*y1*x2+T(5)*y1+T(5)*x1+T(6)*y2+T(6)*
norid[88]=T(3)*T(6)+T(1)*x1+T(1)+T(2)*y1+T(4)+T(5)*y1+T(5)*x1+T(5)+T(6)*y2+T(6)*x1+z*y1^3+y
norid[89]=T(4)*T(6)+T(1)*y1^2+T(1)*y1+T(1)*x1+T(1)+T(2)*y1*x2+T(2)*y1+T(3)*y1*x2+T(4)*y1+T(4)
norid[90]=T(5)*T(6)+T(1)*y1^2+T(1)*y1+T(1)*x1+T(1)+T(2)*y1+T(4)*y1+T(4)+T(5)*y1*x1+T(5)*x1+
norid[91]=T(6)^2+T(1)*y1^3+T(1)*y1*x1^2+T(2)*y1*x2+T(3)*y1*x2+T(4)+T(5)+T(6)*y1*y2+T(6)*y1*
norid[92]=T(1)*T(7)+T(1)*y1+T(1)*x1^2+T(1)*x1+T(2)*y1+T(3)*y1+T(6)+z*y1*y2+y1^2*y2*x2+y1^2*
norid[93]=T(2)*T(7)+T(1)*y1+T(1)*x1^2+T(1)+T(2)*y1*x2+T(3)*y1+T(3)+T(4)+T(6)+y1^4+y1^3+z*y1
norid[94]=T(3)*T(7)+T(1)*y1+T(1)*x1^2+T(1)+T(3)*y1*x2+T(3)*y1+T(3)+T(5)+T(6)+y1^3+z*y1*y2*x
norid[95]=T(4)*T(7)+T(1)*x1^2+T(1)*x1+T(2)*y1*x2+T(2)*y1+T(3)+T(5)*x1+T(5)+T(6)+z^3*x2+z^3+
norid[96]=T(5)*T(7)+T(1)*x1^2+T(1)*x1+T(2)*y1+T(3)*y1*x2+T(3)+T(4)+T(5)*x1+T(6)+z*y1^2*y2+z
norid[97]=T(6)*T(7)+T(1)*y1+T(2)*y1*x2+T(3)*y1*x2+T(3)*y1+T(4)+T(5)*y1+T(5)+T(6)*y2+T(6)*x1
norid[98]=T(7)^2+T(1)*y1*x1+T(1)*x1^3+T(1)*x1^2+T(2)*y1*x2^3+T(2)*y1+T(2)*x2^4+T(2)*x2^2+T(2)
norid[99]=y2^2+y1*x2+y1*x1+y2
norid[100]=y1*y2+y1+x2^2*x1+x2*x1^2
norid[101]=y1^2+y2*x2*x1
norid[102]=y1^3*y2+z^3+z*y1^2+y1^3+y1^2*y2*x2+z*y1*x2*x1+z*y1*x1+y1^2*x2*x1^2+z*y2*x1
norid[103]=y1^4+z*y1^2*y2+y1^3*y2*x2+z^3*x2+z^3*x1+z*y1^2*x2+z*y1^2*x1+y1^3*x2+y1^2*y2*x2^2
norid[104]=z*y1^3+y1^4*x2+z^3*y2+z*y1^2*y2+y1^3*x2^2+y1^3*x2*x1+z*y1*y2*x1+y1^2*y2*x2+z*y1*
norid[105]=y1^5+z*y1^3+y1^4*x1+z^3*y2+z*y1^2*y2*x1+z*y1^2*y2+z^3*x2*x1+z^3*x1^2+z*y1^2*x2*x

```

```

> option(redSB);
> ideal j=std(norid);j;

```

```

j [1]=T(4)*x1+T(5)*y2*x1^2+T(5)*y2*x1+T(5)*x1^2+T(5)*x1+T(6)*y1*x1+T(6)*y2*x1+T(6)*x2^2+T(6)*
j [2]=T(4)*x2^3+T(4)+T(5)*y2*x2^2*x1^2+T(5)*y2*x2^2*x1+T(5)*y2*x1^2+T(5)*y2+T(5)*x2^3*x1+T(5)*
j [3]=T(3)+T(4)*x2+T(4)+T(5)*y2*x1^2+T(5)*y2*x1+T(5)*x2*x1+T(5)*x2+T(6)*y1*x1+T(6)*y2*x1+T(6)*
j [4]=T(2)+T(4)*x2^2+T(4)*x2+T(5)*y2*x2*x1^2+T(5)*y2*x2*x1+T(5)*y2*x1^2+T(5)*y2*x1+T(5)*x2^2
j [5]=T(1)+T(4)+T(5)*y2*x2*x1^2+T(5)*y2*x1^3+T(5)*y2*x2*x1+T(5)*y2*x1+T(5)*x1^2+T(5)*x1+T(5)*
j [6]=y2^2+y1*x2+y1*x1+y2
j [7]=y1*y2+y1+x2^2*x1+x2*x1^2
j [8]=y1^2+y2*x2*x1
j [9]=T(7)*x2^2*x1^2+T(7)*x2*x1^3+T(7)*x2^2*x1+T(7)*x2*x1^2+z^2*y1*x2+z^2*y1*x1+z^2*y2*x2+z^2
j [10]=T(6)*x2+T(6)+T(7)*z*x1^2+T(7)*z*x1+T(7)*y1*x2*x1^3+T(7)*y1*x1^4+T(7)*y1*x2^2*x1+T(7)*
j [11]=T(5)*x1^3+T(5)*x1^2+T(6)+T(7)*z*x1^4+T(7)*z*x1+T(7)*y1*x2*x1^5+T(7)*y1*x1^6+T(7)*y1*x
j [12]=T(5)*x2*x1^2+T(5)*x1^2+T(7)*z*x2*x1^3+T(7)*z*x2*x1+T(7)*y2*x2*x1^5+T(7)*y2*x2*x1^4+T(7)
j [13]=T(5)*x2^3*x1+T(5)*x2^2*x1+T(6)*y2+T(6)*x1+T(7)*z*x1^5+T(7)*z*x1^4+T(7)*z*x2*x1^2+T(7)*
j [14]=T(4)+T(5)*y2*x1^2+T(5)*y2*x1+T(5)*x1^2+T(5)+T(6)*y1*x1+T(6)*y2*x1+T(6)+T(7)*z*x2*x1^2
j [15]=T(7)*y2*x1^2+T(7)*y2*x1+z^2*y1+z^2*y2*x1+z^2*x2*x1+z^2*x1^2+z*y1*x1^2+z*y1*x1+z*y2*x2
j [16]=T(7)*y2*x2*x1+T(7)*y2*x1+T(7)*x2^2*x1+T(7)*x2*x1^2+z^2*y1+z^2*y2*x2+z^2*y2+z^2*x2+z^2
j [17]=T(7)*y1+T(7)*y2*x1+z^2*y2+z^2*x2+z^2*x1+z^2+z*y1*x2+z*y1*x1+z*y2*x2^2+z*y2*x2*x1+z*x2
j [18]=T(7)*z*x1+z^2*y1*x2+z^2*y2+z^2*x2*x1+z^2+y1*x2^2*x1^2+y1*x2^2*x1+y2*x2^3*x1^2+y2*x2^2

```

```

j [19]=T(6)*y2+T(6)*x1+T(7)*x2*x1^2+T(7)*x1^3+z^2*x2*x1+z^2*x1^2+z*y1*x2^2*x1+z*y1*x2*x1^2+z
j [20]=T(6)*y1+T(6)*x1+T(7)*x2*x1^2+T(7)*x1^3+z^3*y1*x2^2*x1+z^3*y1*x1^3+z^3*y1*x2^2+z^3*y1*
j [21]=T(6)*z+z^3*y1*x2^2*x1^3+z^3*y1*x1^5+z^3*y1*x2^3*x1+z^3*y1*x2*x1^3+z^3*y1*x2^3+z^3*y1*
j [22]=T(5)*y2+T(5)*x2^2*x1+T(7)*z+T(7)*y2*x2+T(7)*y2*x1+T(7)*y2+T(7)*x2^2*x1+T(7)*x2*x1+z^3*
j [23]=T(5)*y1+T(5)*x2^2*x1+T(7)*y2*x1+T(7)*x2*x1^2+T(7)*x2*x1+z^3*y1*x2^4+z^3*y1*x1^4+z^3*y
j [24]=T(7)*z*y2+z^2*y1+z^2*y2*x2+z^2*x2^3+z^2*x2^2*x1+y1*x2^4*x1+y1*x2^3*x1^2+y1*x2^3*x1+y1
j [25]=T(5)*z+z^4*y1*x2*x1+z^4*y1*x1^2+z^4*y1*x2+z^4*y1*x1+z^4*y2*x2*x1+z^4*y2*x2+z^4*y2*x1+
j [26]=T(7)^2+T(5)*x2^3+T(5)*x2^2*x1+T(5)*x2^2+T(5)*x2*x1+T(7)*z*x2^3+T(7)*y2*x2^4+T(7)*y2*x
j [27]=T(6)*T(7)+T(7)*z^2*x2+T(7)*x2^2*x1+T(7)*x1^3+z^3*y1*x2^7*x1+z^3*y1*x2^4*x1^4+z^3*y1*x
j [28]=T(5)*T(7)+T(5)*x2*x1+T(5)*x1+T(7)*z^2*x2+T(7)*z+T(7)*y2*x1+z^3*y1*x2^2*x1^7+z^3*y1*x1
j [29]=T(6)^2+T(7)*x2*x1^4+T(7)*x1^5+T(7)*x2^2*x1+T(7)*x2*x1^2+z^5*y1*x2^2*x1^5+z^5*y1*x1^7+
j [30]=T(5)*T(6)+T(7)*z^3*x2^3+T(7)*z^3*x2+T(7)*z^2*x2+T(7)*z^2+T(7)*x2*x1^3+T(7)*x1^4+T(7)*
j [31]=T(5)^2+T(5)*x2^2*x1+T(5)*x2+T(5)*x1+T(5)+T(7)*z^4*x2+T(7)*z^4+T(7)*z^3*x2^3+T(7)*z^3*
j [32]=z^3+z*y1*x2*x1+z*y1*x1+z*y2*x2*x1+z*y2*x1+y1*x2^3*x1+y1*x2^2*x1^2+y2*x2^3*x1^2+y2*x2^
j [33]=T(7)*z^2+z*y1*x2^2+z*y1*x1+z*y2*x2^2*x1+z*y2*x2*x1+z*x2^2*x1+z*x2*x1^2+y1*x2^4*x1+y1*

```

```

> ring r1=2, (f41, f35, f34, f32, f29, f26, f23, z19, y15, y12, x9, u9), wp(41, 35, 34, 32, 29, 26, 23, 19, 15, 1
> map phi=R, f23, f26, f29, f32, f35, f41, f34, z19, y15, y12, x9, u9;
> ideal i1=phi(j);
> option(redSB);
> ideal j1=std(i1);j1;

```

```

j1 [1]=y12^2+y15*x9+y15*u9+y12
j1 [2]=y15*y12+x9^2*u9+x9*u9^2+y15
j1 [3]=y15^2+y12*x9*u9
j1 [4]=f32+f23*x9+f23*u9+f29
j1 [5]=f23*y12+f26*u9+f23*u9+f23
j1 [6]=f35+f26*u9+f23*x9+f29+f23
j1 [7]=f26*y12+f29*x9+f29*u9+f26*x9+f23*u9+f23
j1 [8]=f23*y15+f29*u9+f26*u9
j1 [9]=z19^2+f29*u9+y15*x9*u9+f23*u9+y12*x9*u9+y15*u9+f23+y12*u9
j1 [10]=f29*y12+f23*x9^2+f23*x9*u9+f29*x9+f29*u9+f26*x9+f23*u9+f29+f26+f23
j1 [11]=f26*y15+f23*x9^2+f23*x9*u9+f29*u9+f26*u9
j1 [12]=f41+f23*x9*u9+f23*u9^2
j1 [13]=f23*z19+y15*x9^2*u9+y12*x9^2*u9
j1 [14]=z19*y15*x9+f34*u9+f29*u9+z19*x9*u9+f26*u9+f23*u9+z19*y12+z19
j1 [15]=f29*y15+f26*x9*u9+f23*x9^2+f29*u9+f26*u9+f23*x9
j1 [16]=f26*z19+x9^4*u9+x9^3*u9^2+y15*x9^3+y12*x9^2*u9
j1 [17]=f34*y12+z19*x9^3+z19*x9^2*u9+f23*x9^2+f23*x9*u9+z19*y12*x9+f26*u9+z19*y15+f23*u9+f29
j1 [18]=f23^2+z19*x9^2*u9+f23*x9*u9+f23*u9
j1 [19]=f29*z19+y12*x9^3*u9+y15*x9^3+y12*x9^2*u9+y15*x9^2
j1 [20]=f34*y15+z19*y12*x9^2+f26*x9*u9+f34*u9+f23*x9*u9+z19*x9^2+f23*x9+f23*u9+z19*y12+z19
j1 [21]=f26*f23+z19*y12*x9^2+z19*x9^2*u9+f26*x9*u9+z19*x9^2+f26*u9
j1 [22]=f29*f23+f34*x9*u9+z19*y12*x9^2+f26*x9*u9+f23*x9*u9+z19*y12*x9+f29*u9+z19*x9^2+z19*x9
j1 [23]=f26^2+f34*x9^2+f34*x9*u9+z19*x9^3+f23*x9^2+z19*y12*x9+f29*x9+f29*u9+f26*u9+f23*x9+f2

```

```

j1[24]=f34*z19+f26*x9^2*u9+f23*x9^3+f29*x9*u9+y12*x9^2*u9+f29*u9+f26*u9+y15*x9^2+f23*x9+y12
j1[25]=f29*f26+z19*x9^4+z19*x9^3*u9+f34*x9^2+f23*x9^3+f23*x9^2*u9+z19*x9^3+z19*x9^2*u9+f29*
j1[26]=f34*f23+y12*x9^4*u9+x9^5*u9+x9^4*u9^2+f34*x9*u9+y15*x9^3*u9+y12*x9^3*u9+x9^4*u9+x9^3
j1[27]=f29^2+z19*y12*x9^3+f26*x9^2*u9+f34*x9^2+f34*x9*u9+f23*x9^2*u9+z19*x9^3+f26*x9*u9+f23
j1[28]=f34*f26+y15*x9^5+y15*x9^4*u9+y12*x9^5+z19*x9^4+z19*x9^3*u9+f34*x9*u9+y15*x9^4+f23*x9
j1[29]=f34*f29+x9^6*u9+x9^5*u9^2+z19*y12*x9^3+y12*x9^5+y12*x9^4*u9+z19*x9^4+z19*x9^3*u9+x9^
j1[30]=f34^2+f23*x9^5+f23*x9^4*u9+f29*x9^4+f29*x9^3*u9+x9^6*u9+x9^5*u9^2+f26*x9^4+f26*x9^3*

```

It is clear from the final Gröbner basis given here that f32, f35, f41 were unnecessary, though I would append the unnecessary elements $z19*y12$ and $z19*y15$ to get an $\mathbb{F}_2[x9, u9]$ -module generating set for this as an affine algebra, which is almost an algebra. [At the very least, it must be worthwhile to interreduce the numerators after first reducing them modulo the ideal of the original ring, an alternate way to see that three of the six numerators were unnecessary.]

My output on this example in char 2 is:

```

WT_MATRIX_T= [
  [ 34, 34, 31, 29, 26, 23, 19, 15, 12, 9, 9 ],
  [ 30, 21, 21, 24, 24, 15, 12, 9, 9, 9, 0 ]
]
I=
1 y_34_30*y_9_0+y_34_21*y_9_9+y_34_21*y_9_0+y_31_21+y_19_12
2 y_12_9^2+y_15_9*y_9_9+y_15_9*y_9_0+y_12_9
3 y_15_9^2+y_12_9*y_9_9*y_9_0
4 y_15_9*y_12_9+y_15_9+y_9_9^2*y_9_0+y_9_9*y_9_0^2
5 y_19_12^2+y_29_24*y_9_0+y_23_15
6 y_19_12*y_15_9+y_34_21
7 y_19_12*y_12_9+y_31_21
8 y_23_15^2+y_23_15*y_9_9*y_9_0+y_23_15*y_9_0+y_19_12*y_9_9^2*y_9_0
9 y_23_15*y_19_12+y_19_12*y_9_9*y_9_0+y_19_12*y_9_0+y_15_9*y_9_9^2*y_9_0
+y_12_9*y_9_9^2*y_9_0
10 y_23_15*y_15_9+y_29_24*y_9_0+y_26_24*y_9_0+y_23_15*y_9_0
11 y_23_15*y_12_9+y_26_24*y_9_0+y_23_15*y_9_0+y_23_15
12 y_26_24^2+y_34_30*y_9_9^2+y_29_24*y_9_9^2+y_29_24*y_9_9*y_9_0
+y_29_24*y_9_9+y_29_24*y_9_0+y_26_24*y_9_9^2+y_26_24*y_9_9*y_9_0
+y_26_24*y_9_0+y_26_24*y_23_15*y_9_9^2+y_23_15+y_19_12*y_9_9^2*y_9_0
13 y_26_24*y_23_15+y_31_21*y_9_9^2+y_26_24*y_9_9*y_9_0+y_26_24*y_9_0
+y_19_12*y_9_9^2*y_9_0+y_19_12*y_9_9^2
14 y_26_24*y_19_12+y_31_21*y_9_9+y_31_21+y_19_12*y_9_9*y_9_0
+y_19_12*y_9_9+y_19_12*y_9_0+y_19_12+y_15_9*y_9_9^3
+y_12_9*y_9_9^2*y_9_0+y_9_9^4*y_9_0+y_9_9^3*y_9_0^2
15 y_26_24*y_15_9+y_29_24*y_9_0+y_26_24*y_9_0+y_23_15*y_9_9^2
+y_23_15*y_9_9*y_9_0+y_23_15*y_9_0
16 y_26_24*y_12_9+y_29_24*y_9_9+y_29_24*y_9_0+y_26_24*y_9_9
+y_23_15*y_9_9+y_23_15
17 y_29_24^2+y_34_30*y_9_9^2+y_31_21*y_9_9^3+y_29_24*y_9_9^2

```

$+y_{29_24}y_{9_9}y_{9_0}+y_{29_24}y_{9_9}+y_{29_24}y_{9_0}+y_{26_24}y_{9_9}^2y_{9_0}$
 $+y_{26_24}y_{9_9}^2+y_{26_24}y_{9_0}+y_{26_24}+y_{23_15}y_{9_9}^2y_{9_0}$
 $+y_{23_15}y_{9_9}+y_{23_15}y_{9_0}+y_{23_15}$
18 $y_{29_24}y_{26_24}+y_{34_30}y_{9_9}^2+y_{34_21}y_{9_9}^2+y_{31_21}y_{9_9}^2$
 $+y_{29_24}y_{9_9}^2+y_{29_24}y_{9_9}+y_{26_24}y_{9_9}^2+y_{26_24}y_{9_9}y_{9_0}$
 $+y_{26_24}y_{9_0}+y_{26_24}+y_{23_15}y_{9_9}^3+y_{23_15}y_{9_9}^2y_{9_0}$
 $+y_{23_15}y_{9_0}+y_{23_15}+y_{19_12}y_{9_9}^4+y_{19_12}y_{9_9}^3y_{9_0}$
 $+y_{19_12}y_{9_9}^2$
19 $y_{29_24}y_{23_15}+y_{34_21}y_{9_9}^2+y_{31_21}y_{9_9}^2+y_{29_24}y_{9_9}y_{9_0}$
 $+y_{29_24}y_{9_0}+y_{19_12}y_{9_9}^2$
20 $y_{29_24}y_{19_12}+y_{34_21}y_{9_9}+y_{34_21}+y_{31_21}y_{9_9}+y_{31_21}$
 $+y_{19_12}y_{9_9}+y_{19_12}+y_{15_9}y_{9_9}^3+y_{15_9}y_{9_9}^2y_{9_0}$
 $+y_{15_9}y_{9_9}^2+y_{12_9}y_{9_9}^3y_{9_0}$
21 $y_{29_24}y_{15_9}+y_{26_24}y_{9_9}y_{9_0}+y_{23_15}y_{9_9}^2+y_{23_15}y_{9_9}$
22 $y_{29_24}y_{12_9}+y_{29_24}y_{9_9}+y_{29_24}y_{9_0}+y_{29_24}+y_{26_24}y_{9_9}$
 $+y_{26_24}y_{9_0}+y_{26_24}+y_{23_15}y_{9_9}^2+y_{23_15}y_{9_9}y_{9_0}$
 $+y_{23_15}y_{9_9}+y_{23_15}y_{9_0}+y_{23_15}$
23 $y_{31_21}^2+y_{29_24}y_{9_0}+y_{26_24}y_{9_9}^2y_{9_0}^2+y_{26_24}y_{9_9}y_{9_0}^3$
 $+y_{23_15}y_{9_9}^3y_{9_0}+y_{23_15}y_{9_9}^2y_{9_0}^2+y_{23_15}$
24 $y_{31_21}y_{29_24}+y_{34_21}y_{9_9}^2+y_{34_21}y_{9_9}y_{9_0}+y_{34_21}y_{9_0}$
 $+y_{34_21}+y_{19_12}y_{9_9}^3y_{9_0}+y_{19_12}y_{9_9}^2y_{9_0}^2$
 $+y_{19_12}y_{9_9}^2y_{9_0}+y_{19_12}y_{9_9}y_{9_0}^2+y_{15_9}y_{9_9}^4y_{9_0}$
 $+y_{15_9}y_{9_9}^3y_{9_0}^2+y_{15_9}y_{9_9}^3+y_{15_9}y_{9_9}^2y_{9_0}$
 $+y_{15_9}y_{9_9}^2+y_{12_9}y_{9_9}^3y_{9_0}+y_{9_9}^5y_{9_0}+y_{9_9}^3y_{9_0}^3$
 $+y_{9_9}^4y_{9_0}+y_{9_9}^3y_{9_0}^2$
25 $y_{31_21}y_{26_24}+y_{34_21}y_{9_9}^2+y_{34_21}y_{9_9}y_{9_0}+y_{34_21}y_{9_9}$
 $+y_{34_21}y_{9_0}+y_{31_21}y_{9_9}y_{9_0}+y_{31_21}y_{9_0}+y_{15_9}y_{9_9}^3y_{9_0}$
 $+y_{15_9}y_{9_9}^2y_{9_0}^2+y_{15_9}y_{9_9}^3+y_{12_9}y_{9_9}^4y_{9_0}$
 $+y_{12_9}y_{9_9}^3y_{9_0}^2+y_{12_9}y_{9_9}^2y_{9_0}+y_{9_9}^5y_{9_0}$
 $+y_{9_9}^4y_{9_0}^2$
26 $y_{31_21}y_{23_15}+y_{31_21}y_{9_9}y_{9_0}+y_{31_21}y_{9_0}+y_{15_9}y_{9_9}^3y_{9_0}$
 $+y_{15_9}y_{9_9}^2y_{9_0}^2+y_{15_9}y_{9_9}^2y_{9_0}+y_{12_9}y_{9_9}^2y_{9_0}$
 $+y_{9_9}^4y_{9_0}^2+y_{9_9}^3y_{9_0}^3$
27 $y_{31_21}y_{19_12}+y_{29_24}y_{9_9}y_{9_0}+y_{29_24}y_{9_0}^2+y_{29_24}y_{9_0}$
 $+y_{26_24}y_{9_9}y_{9_0}+y_{26_24}y_{9_0}^2+y_{23_15}y_{9_9}^2y_{9_0}$
 $+y_{23_15}y_{9_9}y_{9_0}^2+y_{23_15}y_{9_9}y_{9_0}+y_{23_15}y_{9_0}^2+y_{23_15}$
28 $y_{31_21}y_{15_9}+y_{34_21}+y_{19_12}y_{9_9}^2y_{9_0}+y_{19_12}y_{9_9}y_{9_0}^2$
29 $y_{31_21}y_{12_9}+y_{34_21}y_{9_9}+y_{34_21}y_{9_0}+y_{31_21}$
30 $y_{34_21}^2+y_{29_24}y_{9_9}^2y_{9_0}^2+y_{29_24}y_{9_9}y_{9_0}^3$
 $+y_{29_24}y_{9_9}y_{9_0}^2+y_{26_24}y_{9_9}^2y_{9_0}^2+y_{26_24}y_{9_9}y_{9_0}^3$
 $+y_{23_15}y_{9_9}^3y_{9_0}^2+y_{23_15}y_{9_9}^2y_{9_0}^3+y_{23_15}y_{9_9}^2y_{9_0}^2$
 $+y_{23_15}y_{9_9}y_{9_0}^3+y_{23_15}y_{9_9}y_{9_0}$
31 $y_{34_21}y_{31_21}+y_{29_24}y_{9_9}^2y_{9_0}^2+y_{29_24}y_{9_9}y_{9_0}^3$
 $+y_{29_24}y_{9_0}+y_{26_24}y_{9_9}y_{9_0}^2+y_{26_24}y_{9_0}$
 $+y_{23_15}y_{9_9}y_{9_0}^2+y_{23_15}y_{9_9}y_{9_0}+y_{23_15}y_{9_0}$
32 $y_{34_21}y_{29_24}+y_{31_21}y_{9_9}^2y_{9_0}+y_{31_21}y_{9_9}y_{9_0}$

$+y_{19_12}y_{9_9}^3y_{9_0}+y_{19_12}y_{9_9}^2y_{9_0}^2+y_{19_12}y_{9_9}^2y_{9_0}$
 $+y_{19_12}y_{9_9}y_{9_0}^2+y_{15_9}y_{9_9}^3y_{9_0}+y_{12_9}y_{9_9}^4y_{9_0}$
 $+y_{12_9}y_{9_9}^3y_{9_0}^2+y_{12_9}y_{9_9}^3y_{9_0}+y_{9_9}^5y_{9_0}^2$
 $+y_{9_9}^4y_{9_0}^3$
33 $y_{34_21}y_{26_24}+y_{34_21}y_{9_9}y_{9_0}+y_{34_21}y_{9_0}+y_{19_12}y_{9_9}^3y_{9_0}$
 $+y_{19_12}y_{9_9}^2y_{9_0}^2+y_{19_12}y_{9_9}^2y_{9_0}+y_{19_12}y_{9_9}y_{9_0}^2$
 $+y_{15_9}y_{9_9}^4y_{9_0}+y_{15_9}y_{9_9}^3y_{9_0}^2+y_{15_9}y_{9_9}^2y_{9_0}$
 $+y_{12_9}y_{9_9}^4y_{9_0}+y_{9_9}^4y_{9_0}^2+y_{9_9}^3y_{9_0}^3$
34 $y_{34_21}y_{23_15}+y_{34_21}y_{9_9}y_{9_0}+y_{34_21}y_{9_0}+y_{15_9}y_{9_9}^2y_{9_0}$
 $+y_{12_9}y_{9_9}^3y_{9_0}^2+y_{9_9}^4y_{9_0}^2+y_{9_9}^3y_{9_0}^3$
35 $y_{34_21}y_{19_12}+y_{29_24}y_{9_0}+y_{26_24}y_{9_9}y_{9_0}^2+y_{26_24}y_{9_0}$
 $+y_{23_15}y_{9_9}^2y_{9_0}+y_{23_15}y_{9_9}y_{9_0}+y_{23_15}y_{9_0}$
36 $y_{34_21}y_{15_9}+y_{31_21}y_{9_9}y_{9_0}$
37 $y_{34_21}y_{12_9}+y_{34_21}y_{19_12}y_{9_9}^2y_{9_0}+y_{19_12}y_{9_9}y_{9_0}^2$
38 $y_{34_30}^2+y_{29_24}y_{9_9}^4+y_{29_24}y_{9_9}^3y_{9_0}+y_{29_24}y_{9_9}^2y_{9_0}^2$
 $+y_{29_24}y_{9_9}y_{9_0}^3+y_{29_24}y_{9_9}^3+y_{29_24}y_{9_9}y_{9_0}^2$
 $+y_{26_24}y_{9_9}^4+y_{26_24}y_{9_9}^3y_{9_0}+y_{26_24}y_{9_9}^2y_{9_0}^2$
 $+y_{26_24}y_{9_9}y_{9_0}^3+y_{26_24}y_{9_9}^2+y_{26_24}y_{9_9}y_{9_0}$
 $+y_{23_15}y_{9_9}^5+y_{23_15}y_{9_9}^4y_{9_0}+y_{23_15}y_{9_9}^3y_{9_0}^2$
 $+y_{23_15}y_{9_9}^2y_{9_0}^3+y_{23_15}y_{9_9}^4+y_{23_15}y_{9_9}^3y_{9_0}$
 $+y_{23_15}y_{9_9}^2y_{9_0}^2+y_{23_15}y_{9_9}y_{9_0}^3+y_{23_15}y_{9_9}^2$
 $+y_{23_15}y_{9_9}y_{9_0}$
39 $y_{34_30}y_{34_21}+y_{29_24}y_{9_9}^3y_{9_0}+y_{29_24}y_{9_9}y_{9_0}^3$
 $+y_{26_24}y_{9_9}^3y_{9_0}+y_{26_24}y_{9_9}y_{9_0}^3+y_{23_15}y_{9_9}^4y_{9_0}$
 $+y_{23_15}y_{9_9}^2y_{9_0}^3+y_{23_15}y_{9_9}^3y_{9_0}+y_{23_15}y_{9_9}y_{9_0}^3$
40 $y_{34_30}y_{31_21}+y_{29_24}y_{9_9}^3y_{9_0}+y_{29_24}y_{9_9}y_{9_0}^3$
 $+y_{23_15}y_{9_9}^3+y_{23_15}y_{9_9}y_{9_0}^2$
41 $y_{34_30}y_{29_24}+y_{34_30}y_{9_9}+y_{34_30}y_{31_21}y_{9_9}^3$
 $+y_{31_21}y_{9_9}^2y_{9_0}+y_{31_21}y_{9_9}^2+y_{31_21}y_{9_9}y_{9_0}$
 $+y_{19_12}y_{9_9}^4+y_{19_12}y_{9_9}^2y_{9_0}^2+y_{19_12}y_{9_9}^2y_{9_0}$
 $+y_{19_12}y_{9_9}y_{9_0}^2+y_{19_12}y_{9_9}^2+y_{19_12}y_{9_9}y_{9_0}$
 $+y_{12_9}y_{9_9}^5+y_{12_9}y_{9_9}^3y_{9_0}^2+y_{12_9}y_{9_9}^4$
 $+y_{12_9}y_{9_9}^3y_{9_0}+y_{9_9}^6y_{9_0}+y_{9_9}^4y_{9_0}^3+y_{9_9}^5$
 $+y_{9_9}^3y_{9_0}^2+y_{9_9}^4+y_{9_9}^3y_{9_0}$
42 $y_{34_30}y_{26_24}+y_{34_30}y_{9_9}+y_{34_30}y_{34_21}y_{9_9}^2$
 $+y_{34_21}y_{9_9}y_{9_0}+y_{34_21}y_{9_9}+y_{34_21}y_{9_0}+y_{31_21}y_{9_9}$
 $+y_{31_21}y_{19_12}y_{9_9}^4+y_{19_12}y_{9_9}^2y_{9_0}^2+y_{19_12}y_{9_9}^3$
 $+y_{19_12}y_{9_9}y_{9_0}^2+y_{19_12}y_{9_9}+y_{19_12}y_{15_9}y_{9_9}^5$
 $+y_{15_9}y_{9_9}^3y_{9_0}^2+y_{12_9}y_{9_9}^5+y_{12_9}y_{9_9}^4y_{9_0}$
 $+y_{12_9}y_{9_9}^4+y_{12_9}y_{9_9}^3y_{9_0}+y_{9_9}^5y_{9_0}+y_{9_9}^3y_{9_0}^3$
 $+y_{9_9}^5+y_{9_9}^4y_{9_0}+y_{9_9}^4+y_{9_9}^3y_{9_0}$
43 $y_{34_30}y_{23_15}+y_{34_21}y_{9_9}^2+y_{34_21}y_{9_9}y_{9_0}+y_{34_21}y_{9_9}$
 $+y_{34_21}y_{9_0}+y_{31_21}y_{9_9}+y_{31_21}y_{19_12}y_{9_9}+y_{19_12}$
 $+y_{12_9}y_{9_9}^4y_{9_0}+y_{12_9}y_{9_9}^3y_{9_0}^2+y_{9_9}^5y_{9_0}$
 $+y_{9_9}^3y_{9_0}^3+y_{9_9}^4y_{9_0}+y_{9_9}^3y_{9_0}^2$
44 $y_{34_30}y_{19_12}+y_{26_24}y_{9_9}^2y_{9_0}+y_{26_24}y_{9_9}y_{9_0}^2$

$$\begin{aligned}
& +y_{23_15}y_{9_9}^3+y_{23_15}y_{9_9}^2y_{9_0} \\
45 & y_{34_30}y_{15_9}+y_{31_21}y_{9_9}^2+y_{31_21}y_{9_9}y_{9_0}+y_{19_12}y_{9_9}^2 \\
& +y_{19_12}y_{9_9}y_{9_0} \\
46 & y_{34_30}y_{12_9}+y_{19_12}y_{9_9}^3+y_{19_12}y_{9_9}y_{9_0}^2
\end{aligned}$$

$$\begin{aligned}
\text{delta} = & x_{9_9}^3x_{9_0}^3+x_{9_9}^2x_{9_0}^4+x_{9_9}^3x_{9_0}^2+x_{9_9}x_{9_0}^4+ \\
& x_{9_9}^2x_{9_0}^2+x_{9_9}x_{9_0}^3
\end{aligned}$$

phi=

$$\begin{aligned}
1 & x_{19_12}x_{15_9}x_{9_9}^4x_{9_0}^2+x_{19_12}x_{15_9}x_{9_9}^2x_{9_0}^4+ \\
& x_{19_12}x_{15_9}x_{9_9}^4x_{9_0}+x_{19_12}x_{15_9}x_{9_9}^3x_{9_0}^2+ \\
& x_{19_12}x_{15_9}x_{9_9}^2x_{9_0}^3+x_{19_12}x_{15_9}x_{9_9}x_{9_0}^4+ \\
& x_{19_12}x_{12_9}x_{9_9}^3x_{9_0}^2+x_{19_12}x_{12_9}x_{9_9}^2x_{9_0}^3+ \\
& x_{19_12}x_{15_9}x_{9_9}^3x_{9_0}+x_{19_12}x_{15_9}x_{9_9}x_{9_0}^3+ \\
& x_{19_12}x_{12_9}x_{9_9}^3x_{9_0}+x_{19_12}x_{12_9}x_{9_9}x_{9_0}^3+ \\
& x_{19_12}x_{9_9}^3x_{9_0}^2+x_{19_12}x_{9_9}^2x_{9_0}^3+ \\
& x_{19_12}x_{12_9}x_{9_9}^2x_{9_0}+x_{19_12}x_{12_9}x_{9_9}x_{9_0}^2+ \\
& x_{19_12}x_{9_9}^3x_{9_0}+x_{19_12}x_{9_9}x_{9_0}^3+x_{19_12}x_{9_9}^2x_{9_0}+ \\
& x_{19_12}x_{9_9}x_{9_0}^2 \\
2 & x_{19_12}x_{15_9}x_{9_9}^3x_{9_0}^3+x_{19_12}x_{15_9}x_{9_9}^2x_{9_0}^4+ \\
& x_{19_12}x_{15_9}x_{9_9}x_{9_0}^3x_{9_0}^2+x_{19_12}x_{15_9}x_{9_9}x_{9_0}^4+ \\
& x_{19_12}x_{15_9}x_{9_9}^2x_{9_0}^2+x_{19_12}x_{15_9}x_{9_9}x_{9_0}^3 \\
3 & x_{19_12}x_{12_9}x_{9_9}^3x_{9_0}^3+x_{19_12}x_{12_9}x_{9_9}^2x_{9_0}^4+ \\
& x_{19_12}x_{12_9}x_{9_9}x_{9_0}^2+x_{19_12}x_{12_9}x_{9_9}x_{9_0}^4+ \\
& x_{19_12}x_{12_9}x_{9_9}^2x_{9_0}^2+x_{19_12}x_{12_9}x_{9_9}x_{9_0}^3 \\
4 & x_{19_12}^2x_{9_9}^3x_{9_0}^2+x_{19_12}^2x_{9_9}^2x_{9_0}^3+ \\
& x_{19_12}^2x_{9_9}^3x_{9_0}+x_{19_12}^2x_{9_9}x_{9_0}^3+ \\
& x_{19_12}^2x_{12_9}x_{9_9}x_{9_0}+x_{19_12}^2x_{15_9}x_{9_9} \\
& x_{19_12}^2x_{15_9}x_{9_0}+x_{19_12}^2x_{9_9}x_{9_0}+x_{19_12}^2x_{12_9}+ \\
& x_{19_12}^2 \\
5 & x_{19_12}^2x_{15_9}x_{9_9}^2x_{9_0}+x_{19_12}^2x_{15_9}x_{9_9}x_{9_0}^2+ \\
& x_{19_12}^2x_{12_9}x_{9_9}^2x_{9_0}+x_{19_12}^2x_{9_9}^3x_{9_0}+ \\
& x_{19_12}^2x_{9_9}^2x_{9_0}^2+x_{19_12}^2x_{15_9}x_{9_9}x_{9_0}+ \\
& x_{19_12}^2x_{15_9}x_{9_0}^2+x_{19_12}^2x_{12_9}x_{9_9}x_{9_0}+ \\
& x_{19_12}^2x_{9_9}^2x_{9_0}+x_{19_12}^2x_{15_9}x_{9_9}+x_{19_12}^2x_{15_9}x_{9_0}+ \\
& x_{19_12}^2x_{12_9}x_{9_9}+x_{19_12}^2x_{9_9}x_{9_0}+x_{19_12}^2x_{12_9}+ \\
& x_{19_12}^2x_{9_0}+x_{19_12}^2 \\
6 & x_{19_12}^2x_{12_9}x_{9_9}x_{9_0}^2+x_{19_12}^2x_{9_9}^2x_{9_0}^2+ \\
& x_{19_12}^2x_{9_9}x_{9_0}^3+x_{19_12}^2x_{15_9}x_{9_9}x_{9_0}+ \\
& x_{19_12}^2x_{15_9}x_{9_0}^2+x_{19_12}^2x_{9_9}x_{9_0}^2+x_{19_12}^2x_{12_9}x_{9_9} \\
& +x_{19_12}^2x_{9_0} \\
7 & x_{19_12}x_{9_9}^3x_{9_0}^3+x_{19_12}x_{9_9}^2x_{9_0}^4+x_{19_12}x_{9_9}^3x_{9_0}^2+ \\
& x_{19_12}x_{9_9}x_{9_0}^4+x_{19_12}x_{9_9}^2x_{9_0}^2+x_{19_12}x_{9_9}x_{9_0}^3 \\
8 & x_{15_9}x_{9_9}^3x_{9_0}^3+x_{15_9}x_{9_9}^2x_{9_0}^4+x_{15_9}x_{9_9}^3x_{9_0}^2+ \\
& x_{15_9}x_{9_9}x_{9_0}^4+x_{15_9}x_{9_9}^2x_{9_0}^2+x_{15_9}x_{9_9}x_{9_0}^3 \\
9 & x_{12_9}x_{9_9}^3x_{9_0}^3+x_{12_9}x_{9_9}^2x_{9_0}^4+x_{12_9}x_{9_9}^3x_{9_0}^2+
\end{aligned}$$

```

      x_12_9*x_9_9*x_9_0^4+x_12_9*x_9_9^2*x_9_0^2+x_12_9*x_9_9*x_9_0^3
10 x_9_9^4*x_9_0^3+x_9_9^3*x_9_0^4+x_9_9^4*x_9_0^2+x_9_9^2*x_9_0^4+
      x_9_9^3*x_9_0^2+x_9_9^2*x_9_0^3
11 x_9_9^3*x_9_0^4+x_9_9^2*x_9_0^5+x_9_9^3*x_9_0^3+x_9_9*x_9_0^5+
      x_9_9^2*x_9_0^3+x_9_9*x_9_0^4

```

```

psi=
1 f_19_12
2 f_15_9
3 f_12_9
4 f_9_9
5 f_9_0

```

0.480 seconds

Here is a char 0 version in SINGULAR.

The weighted version in SINGULAR 3-1-0 clearly only needs the functions T(2),T(8),T(9) to describe the integral closure over the input ring, as seen by looking at a standard (Gröbner) basis for the output:

```

                                SINGULAR                               /
A Computer Algebra System for Polynomial Computations / version 3-1-0
                                                    0<
by: G.-M. Greuel, G. Pfister, H. Schoenemann \ Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern \
> LIB "normal.lib";
> ring r=0, (z,y1,y2,x2,x1), wp(19,15,12,9,9);
> poly p1=y1^2-y2*x2*x1;
> poly p2=y1*y2-y1-x2*x1*(x2+x1);
> poly p3=y2^2-y2-y1*(x2+x1);
> poly p4=z^3-z*(y1+y2)*(x2+1)*x1-(y2*(x2*x1+1)+y1*(x2+x1))*x2^2*x1;
> ideal i=p1,p2,p3,p4;

> list nor=normal(i);
> nor;
[1]:
  [1]:
// characteristic : 0
// number of vars : 14
//      block   1 : ordering dp
//                : names   T(1) T(2) T(3) T(4) T(5) T(6) T(7) T(8) T(9)
//      block   2 : ordering wp
//                : names   z y1 y2 x2 x1
//                : weights 19 15 12  9  9
//      block   3 : ordering C
[2]:

```

```

[1]:
_[1]=-z^3+z*y1*x2*x1+z*y1*x1+z*y2*x1
_[2]=z^2*y1*x2
_[3]=z^3*x2+z^3*x1-z^3-z*y1*x1^2-z*y2*x1^2+z*y1*x1+z*y2*x1
_[4]=z^2*y1*y2-z^2*y1
_[5]=z^3*y2-z^3-z*y1*y2*x1-z*y1*x1^2+z*y1*x1
_[6]=z^3*y1-z*y1*y2*x1
_[7]=z^4-z^2*y1*x1-z^2*y2*x1
_[8]=z*y1*y2*x2^2-z^3-z*y1*x2^2+z*y1*x1+z*y2*x1
_[9]=z^2*y2*x2^2-z^2*x2^2-z^2*x2*x1
_[10]=y1^2
> def R=nor[1][1];
> setring R;
> normap;
normap[1]=z
normap[2]=y1
normap[3]=y2
normap[4]=x2
normap[5]=x1
> norid;
norid[1]=T(2)*y2-T(2)-T(4)*x2
norid[2]=-T(1)*y2+T(1)+T(3)*y2-T(3)-T(5)*x2-T(5)*x1
norid[3]=T(2)*y1-T(4)-T(9)*x1
norid[4]=-T(1)*y1+T(3)*y1-T(5)*y2
norid[5]=T(1)*z-T(2)*x1+T(7)
norid[6]=-T(1)*y2+T(1)+T(2)*z-T(5)-T(6)*x2
norid[7]=-T(1)*y2+T(1)+T(4)*z+T(5)*y2-T(5)-T(6)*y2-T(6)*x2-T(6)*x1+T(6)
norid[8]=-T(2)*x1+T(5)*z-T(7)*y2+T(7)
norid[9]=-T(4)*y2+T(4)+T(6)*z-T(7)*y1+T(9)*x1
norid[10]=-T(2)*x1+T(7)+T(8)*z-T(9)*y1
norid[11]=T(1)*x2*x1+T(1)*x1^2-T(1)*y1-T(1)*x1+T(3)*x1+T(5)*y1-T(6)*y2
norid[12]=T(2)*x2*x1+T(2)*x1^2-T(4)*y1
norid[13]=T(1)*x1^2-T(1)*y1-T(1)*x1+T(3)*x2*x1+T(3)*x1-T(6)*y2
norid[14]=T(1)*y2*x1+T(1)*x1^2-T(1)*y1-2*T(1)*x1+T(3)*x1+T(5)*x2*x1+T(5)*y1+T(5)*x1-T(6)*y1
norid[15]=-T(2)*x1^2+T(3)*z*x1-T(4)*y2+T(4)-T(7)*x2*x1-T(7)*x1^2+T(7)*x1+T(9)*x1
norid[16]=T(2)*x2^2-T(2)*x1^2+T(2)*x1-T(3)*z+T(4)*y1+T(7)*x2+T(7)*x1-T(7)-T(9)*y2
norid[17]=-T(2)*x1+T(4)*x2^2-T(9)*y1
norid[18]=-T(1)*x1^2+T(1)*y1-T(3)*x1+T(5)*x2^2-T(5)*y1+T(6)*y2+T(8)*x1-T(9)*z-z*x1
norid[19]=T(1)*y2*x2-2*T(1)*x2+T(1)-T(3)+T(5)*x2-T(8)*x1+z
norid[20]=T(4)*y2*x2-T(4)*x2-T(4)*x1-T(9)*x2*x1-T(9)*x1^2
norid[21]=T(1)*y1*x1-T(1)*x2^2+T(1)*x1^2-T(1)*y2-T(1)*x1+2*T(1)-T(3)*z-T(3)*x2+T(3)*x1-T(3)*
norid[22]=T(1)*y1*x2+T(1)*x2^2-T(1)*x1^2-T(1)*x2+T(1)*x1+T(3)*z+T(3)*x2-T(3)*x1-T(5)*y1+T(6)
norid[23]=T(4)*y1*x2-T(4)*y2+T(4)-T(9)*y2*x1+T(9)*x1
norid[24]=-T(1)*x2^2+T(1)*x1^2-T(1)*y2+T(1)*x2-T(1)*x1+T(1)-T(3)*z-T(3)*x2+T(3)*x1-T(5)*y1*
norid[25]=-T(1)*x2^2+T(1)*x1^2-2*T(1)*y1+T(1)*y2+T(1)*x2-T(1)-T(3)*x2-T(4)*x2*x1-T(4)*y1+T(6)
norid[26]=T(2)*x1^2+T(3)*z*x2-T(4)*y1-T(4)-T(7)*x2^2-T(7)*x2*x1+T(7)*x2-T(9)*x1

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norid[27]=T(1)*y1*y2+T(1)*x2^2-T(1)*x1^2-T(1)*y1-T(1)*x2+T(1)*x1+T(3)*z+T(3)*x2-T(3)*x1+T(5)
 norid[28]=-T(1)*x2-T(1)*x1+T(1)-T(3)+z*y2
 norid[29]=T(1)*y1^2+2*T(1)*x1^2-2*T(1)*y1-T(1)*x1+T(3)*x1+T(5)*x2*x1+2*T(5)*y1-2*T(6)*y2-T(6)
 norid[30]=T(1)*x2^2-T(1)*y1-T(1)*x2-T(1)+T(3)*z+T(3)*x2+T(4)*y2+T(4)*x2+T(4)*x1-T(4)+T(5)*y1
 norid[31]=-T(1)*y2+T(1)-T(5)+z*y1
 norid[32]=T(2)*x1-T(3)*z+T(7)*x2+T(7)*x1-T(7)+z^2
 norid[33]=T(1)*x1^2-2*T(1)*y1-T(1)*x1+T(3)*z^2+T(3)*x1+T(5)*x2*x1+2*T(5)*y1-T(6)*x2*x1-2*T(6)
 norid[34]=T(1)*x1^2-2*T(1)*y1-T(1)*x1+T(3)*x1-T(4)*y1^2+T(4)*x1^2-T(4)*y2+T(5)*x2*x1+2*T(5)
 norid[35]=-T(1)-y1*y2+x2*x1^2-y2*x2-z+y1
 norid[36]=T(2)*x1^2-T(4)*y1*y2+T(4)*x2*x1^2+T(4)*y1+T(9)*y1*x1
 norid[37]=T(1)*x1^3-T(1)*y1*x1-T(1)*x2^2+T(1)*x1^2-T(1)*y1+T(1)*x2-T(1)*x1-T(3)*z+T(3)*x1^2
 norid[38]=T(1)*x1^3-T(1)*y1*x1-T(1)*x1^2-T(1)*y1+T(1)*y2+T(1)*x2-T(1)*x1-2*T(1)+T(2)*x1+T(3)
 norid[39]=-T(1)*x1^2+2*T(1)*y1+T(1)*x1-T(3)*x1+T(4)*x2*x1+T(4)*y2-T(5)*x2*x1-2*T(5)*y1+T(6)
 norid[40]=T(1)*y1*x1^2+T(1)*y1*x1+T(1)*x2^2+T(1)*x1^2-T(1)*y1-T(1)*y2-T(1)*x2+T(1)-T(2)*x1^2
 norid[41]=-T(1)*x1^3+2*T(1)*y1*x1-T(1)*x2^2+2*T(1)*x1^2+T(1)*y1-3*T(1)*y2-T(1)*x1+4*T(1)-T(5)
 norid[42]=T(1)*y1-T(1)*y2+T(1)+T(5)*y2-T(5)-T(6)*x2-T(6)*x1+T(6)+z*x1^2-z*x1
 norid[43]=-T(1)*y1*x1+T(1)*x1^2-T(1)*y1+2*T(1)*y2-2*T(1)+T(2)*x1^2-T(4)*y1*x1-T(4)*y2*x1+T(4)
 norid[44]=-T(1)*x1^3+T(1)*x1^2-T(1)*y1-T(1)*x2+T(1)*x1+T(1)+T(2)*x1^2-T(2)*x1-T(3)*x1^2-T(3)
 norid[45]=-T(1)*y1*x1+T(1)*x2^2-T(1)*x1^2-2*T(1)*y1+2*T(1)*y2-T(1)*x2-2*T(1)+T(3)*z+T(3)*x2
 norid[46]=T(1)*y1*x1-2*T(1)*x2^2+2*T(1)*x1^2+2*T(1)*x2-2*T(1)*x1-2*T(3)*z-2*T(3)*x2+2*T(3)*
 norid[47]=-T(1)*x2^2-T(1)*x1^2+3*T(1)*y1+T(1)*x2-T(3)*z-T(3)*x2+T(4)*y1^2-T(4)*y2^2+2*T(4)*
 norid[48]=T(1)*x2^3+T(1)*x1^3-T(1)*y1*x1-3*T(1)*x2^2+T(1)*x1^2+2*T(1)*x2-3*T(1)*x1-2*T(3)*z
 norid[49]=-2*T(1)*y1+T(1)*y2-T(1)-T(4)*x2*x1-T(4)*y1+T(5)*y1-T(5)*y2+T(5)+T(6)*x2^3+T(6)*x2
 norid[50]=T(1)*x1^2-T(1)*y1-2*T(1)*x1+T(3)*x1+T(5)*x2*x1+T(5)*y1+T(5)*x1+T(6)*y2*x2^2-T(6)*
 norid[51]=T(7)*y2*x2^2-T(7)*x2^2-T(7)*x2*x1-T(9)*z^2+T(9)*y1*x1+T(9)*y2*x1
 norid[52]=T(1)*x2^2-T(1)*x1^2+2*T(1)*y1-T(1)*y2+T(1)*x1+T(1)+T(3)*x2+T(4)*x2*x1+T(4)*y1-T(5)
 norid[53]=T(1)*y2+T(1)*x2+T(1)*x1-T(1)+T(3)+y1^2*x2+y1*x2^2+y1*x2*x1+y2*x2
 norid[54]=-T(1)*x1^3+T(1)*y2^2-2*T(1)*y1*x1+T(1)*x2^2-2*T(1)*y2-T(1)*x2+T(1)+T(3)*z+T(3)*x2
 norid[55]=-T(4)*y2^2+T(4)*y1*x1+2*T(4)*y2-T(4)+T(7)*y1^2*x2-T(7)*y1*y2+T(7)*y2^2-T(7)*y1*x2
 norid[56]=T(1)*y1-T(1)*x2-T(3)*x1-T(3)-T(5)*y1-T(5)*y2+T(5)+T(6)*y2+T(6)*x2+T(6)*x1+y1^3+y1
 norid[57]=3*T(1)*y1*x1-2*T(1)*x2^2-T(1)*x1^2+3*T(1)*y1-3*T(1)*y2+2*T(1)*x2+T(1)*x1+3*T(1)+T
 norid[58]=-T(1)*x1^4-T(1)*x1^3-T(1)*y2^2+2*T(1)*y1*x1-4*T(1)*x2^2+8*T(1)*x1^2-2*T(1)*y1-T(1)
 norid[59]=-T(1)*x1^3+T(1)*y2^2-T(1)*y1*x1+T(1)*x2^2-3*T(1)*x1^2+T(1)*y1-T(1)*x2+T(1)
 norid[60]=-T(1)*x1^3-4*T(1)*x2^2+3*T(1)*y1-2*T(1)*y2+3*T(1)*x2-T(1)*x1+3*T(1)+T(2)*x1^2-T(2)
 norid[61]=-T(1)*y1*x1+2*T(1)*x2^2-2*T(1)*x1^2-2*T(1)*x2+2*T(1)*x1+2*T(3)*z+2*T(3)*x2-2*T(3)
 norid[62]=-T(1)*x1^3+6*T(1)*y1*x1-2*T(1)*x2^2-4*T(1)*x1^2+14*T(1)*y1+3*T(1)*y2+2*T(1)*x2+6*
 norid[63]=-T(1)*x1^3+23*T(1)*y2^2-12*T(1)*y1*x1+30*T(1)*x2^2-72*T(1)*x1^2+16*T(1)*y1-37*T(1)
 norid[64]=6*T(1)*x1^3-4*T(1)*y2^2-2*T(1)*y1*x1+3*T(1)*x2^2-8*T(1)*x1^2+15*T(1)*y1+27*T(1)*y
 norid[65]=-T(1)*x1^3+6*T(1)*y2^2+2*T(1)*y1*x1+12*T(1)*x2^2-27*T(1)*x1^2+8*T(1)*y1-7*T(1)*y2
 norid[66]=-9*T(1)*x1^3-15*T(1)*y2^2+7*T(1)*y1*x1-17*T(1)*x2^2+83*T(1)*x1^2-31*T(1)*y1-8*T(1)
 norid[67]=T(1)^2-2*T(1)*y1*x2*x1-2*T(1)*y1*y2+2*T(1)*x2*x1^2+3*T(1)*x1^3-T(1)*y1*x2+T(1)*y1
 norid[68]=T(1)*T(2)-T(1)*y1^3-T(1)*y1*y2*x1^2-T(1)*x1^4-2*T(1)*y1*x2*x1-3*T(1)*y1*x1^2+T(1)
 norid[69]=T(2)^2+T(1)*y1*x1^2+T(1)*z*x2-T(1)*z*x1+T(1)*x2*x1^2+T(1)*x1^3-T(1)*y1*x1+T(1)*z-
 norid[70]=T(1)*T(3)-T(1)*y1*x2*x1-T(1)*y1*y2+T(1)*x2*x1^2+2*T(1)*x1^3-T(1)*y1*x2+T(1)*y1*x1
 norid[71]=T(2)*T(3)-T(1)*y1^3-T(1)*y1*y2*x1^2-T(1)*x1^4-2*T(1)*y1*x2*x1-3*T(1)*y1*x1^2+T(1)
 norid[72]=T(3)^2+T(1)*z*x2+T(1)*z*x1+T(1)*x1^3-T(1)*y1*x2+T(1)*y1*x1-T(1)*z+5*T(1)*x1^2-2*T

norid[73]=T(1)*T(4)-T(1)*y1*y2*x2-T(1)*y1*y2*x1-T(1)*x1^3-T(1)*y1*x2-3*T(1)*y1*x1+T(1)*x2^2
 norid[74]=T(2)*T(4)+T(1)*y1*x2*x1+T(1)*y1*y2-T(1)*x2*x1^2-T(1)*x1^3+T(1)*y1*x1-T(1)*y2*x1-T
 norid[75]=T(3)*T(4)+T(1)*x1^3-2*T(1)*y1*x1-T(1)*x1^2-T(1)*y1+3*T(1)*x2-4*T(1)*x1-3*T(1)-T(2)
 norid[76]=T(4)^2+T(1)*z*x1^3-2*T(1)*z*y1*x1+2*T(1)*y1*x2*x1^2-T(1)*z*x1^2+2*T(1)*y1*y2*x1-2
 norid[77]=T(1)*T(5)-T(1)*z*x1^2+2*T(1)*z*y1-4*T(1)*y1*x2*x1+T(1)*y1^2+T(1)*z*x1-4*T(1)*y1*y
 norid[78]=T(2)*T(5)-T(1)*y1^3-T(1)*y1*y2*x1^2+T(1)*y1*y2*x2+T(1)*y1*y2*x1-T(1)*x1^4-2*T(1)*
 norid[79]=T(3)*T(5)-T(1)*z*x1^2+2*T(1)*z*y1-4*T(1)*y1*x2*x1+T(1)*z*y2+T(1)*y1^2+T(1)*z*x1-4
 norid[80]=T(4)*T(5)+T(1)*z^2-T(1)*y1*y2*x2-T(1)*y1*y2*x1+T(1)*z*y1+T(1)*z*x2+T(1)*z*x1-T(1)
 norid[81]=T(5)^2-T(1)*z*x1^2+3*T(1)*z*y1-3*T(1)*y1*x2*x1+T(1)*y1^2+T(1)*z*x2+T(1)*z*x1-3*T
 norid[82]=T(1)*T(6)+T(1)*z*y1*y2*x2+T(1)*z*y1*y2*x1+T(1)*z*x1^3+T(1)*y1^3+T(1)*y1*y2*x1^2+T
 norid[83]=T(2)*T(6)-T(1)*z*y1*x1+T(1)*z^2-T(1)*z*x2*x1-T(1)*z*x1^2+T(1)*z*y1+T(1)*z*x2+2*T
 norid[84]=T(3)*T(6)+T(1)*z*y1*y2*x2+T(1)*z*y1*y2*x1+T(1)*z*x1^3+T(1)*y1^3+T(1)*y1*y2*x1^2+T
 norid[85]=T(4)*T(6)-T(1)*x1^4-T(1)*z*y1+2*T(1)*y1*x1^2-T(1)*z*x2-T(1)*z*x1+T(1)*x1^3+T(1)*y
 norid[86]=T(5)*T(6)+T(1)*z*y1*y2*x2+T(1)*z*y1*y2*x1+T(1)*z*x1^3+T(1)*y1^3+T(1)*y1*y2*x1^2+T
 norid[87]=T(6)^2+T(1)*z^2*y1+T(1)*z^2*x2+T(1)*z^2*x1+T(1)*z*x1^3-2*T(1)*z*y1*x1+T(1)*y1*x2*
 norid[88]=T(1)*T(7)-1/2*T(1)*z*x1^3-T(1)*y1^3-T(1)*y1*y2*x1^2+1/2*T(1)*z*y1*x2-1/2*T(1)*z*y
 norid[89]=T(2)*T(7)+T(1)*z*y1^3+T(1)*z*y1*y2*x1^2+T(1)*z*x1^4+T(1)*z*y1*x2*x1+3*T(1)*z*y1*x
 norid[90]=T(3)*T(7)-1/2*T(1)*z*x1^3-T(1)*y1^3-T(1)*y1*y2*x1^2+1/2*T(1)*z*y1*x2-1/2*T(1)*z*y
 norid[91]=T(4)*T(7)+T(1)*z*y1*y2*x2+T(1)*z*y1*y2*x1+T(1)*z*x1^3+T(1)*y1^3+T(1)*y1*y2*x1^2+T
 norid[92]=T(5)*T(7)+T(1)*z^2*x1^2-2*T(1)*z^2*y1-T(1)*z^2*x1-1/2*T(1)*z*x1^3-T(1)*y1^3-T(1)*
 norid[93]=T(6)*T(7)-T(1)*z^2*y1*y2*x2-T(1)*z^2*y1*y2*x1-T(1)*z^2*x1^3-T(1)*z*y1^3-T(1)*z*y1
 norid[94]=T(7)^2+2*T(1)*z*y1^3+2*T(1)*z*y1*y2*x1^2-8*T(1)*y1^4-4*T(1)*y1^2*y2*x2^2-4*T(1)*y
 norid[95]=T(1)*T(8)-2*T(1)*y1*x2*x1+T(1)*z*x2-2*T(1)*y1*y2+2*T(1)*x2*x1^2+2*T(1)*x1^3-T(1)*
 norid[96]=T(2)*T(8)-T(1)*y1^3-T(1)*y1*y2*x1^2-T(1)*x1^4-2*T(1)*y1*x2*x1-3*T(1)*y1*x1^2+T(1)
 norid[97]=T(3)*T(8)-T(1)*y1*y2*x2-T(1)*y1*y2*x1-T(1)*y1*x2*x1+T(1)*z*x2-T(1)*y1*y2+T(1)*x2*
 norid[98]=T(4)*T(8)-T(1)*z*y1*y2*x2-T(1)*z*y1*y2*x1-T(1)*z*x1^3-T(1)*y1^3-T(1)*y1*y2*x1^2-T
 norid[99]=T(5)*T(8)+T(1)*z*x2^2-T(1)*z*x1^2+2*T(1)*z*y1-4*T(1)*y1*x2*x1+T(1)*z*x2+T(1)*z*x1
 norid[100]=T(6)*T(8)+T(1)*z*y1*y2*x2+T(1)*z*y1*y2*x1+T(1)*z*x1^3+T(1)*y1^3+T(1)*y1*y2*x1^2+
 norid[101]=T(7)*T(8)-1/2*T(1)*z*x1^3-2*T(1)*y1^3-2*T(1)*y1*y2*x1^2+1/2*T(1)*z*y1*x2-1/2*T(1)
 norid[102]=T(8)^2+1/2*T(1)*z*x1^3-T(1)*y1^3-T(1)*y1*y2*x1^2-1/2*T(1)*z*y1*x2+1/2*T(1)*z*y1*
 norid[103]=T(1)*T(9)+T(1)*z*y1+T(1)*z*x2+T(1)*z*x1-2*T(1)*x1^3+4*T(1)*y1*x1-T(1)*y2*x2-T(1)
 norid[104]=T(2)*T(9)+T(1)*y1*x2*x1+T(1)*y1*y2-T(1)*x2*x1^2-T(1)*x1^3-T(1)*y2*x1+2*T(1)*x1^2
 norid[105]=T(3)*T(9)-T(1)*y1*y2*x2-T(1)*y1*y2*x1+T(1)*z*y1+T(1)*y2*x2^2+T(1)*z*x2+T(1)*z*x1
 norid[106]=T(4)*T(9)+T(1)*z*x2^2-T(1)*y1^2+T(1)*z*x2-T(1)*y1*x2+T(1)*x2*x1+T(1)*x1^2-T(1)*x
 norid[107]=T(5)*T(9)+T(1)*y1*x2^2-2*T(1)*x1^3+4*T(1)*y1*x1-T(1)*y2*x2+2*T(1)*x1^2+T(1)*y1-7
 norid[108]=T(6)*T(9)-T(1)*y1^3-T(1)*y1*y2*x1^2+T(1)*y1*y2*x2+T(1)*y1*y2*x1-T(1)*x1^4-2*T(1)
 norid[109]=T(7)*T(9)-T(1)*z^2*y1-T(1)*z^2*x2-T(1)*z^2*x1+T(1)*z*x1^3-2*T(1)*z*y1*x1+3*T(1)*
 norid[110]=T(8)*T(9)-T(1)*y1*x2^3+T(1)*y1*y2*x1-T(1)*x2*x1^3+T(1)*z*y1-T(1)*y1*x2*x1+T(1)*y
 norid[111]=T(9)^2+T(1)*y1*x2^5-T(1)*y1^3*x2+T(1)*x2^6+T(1)*y1^3*x1-T(1)*y1*y2*x1^3+T(1)*x2*
 norid[112]=y2^2-y1*x2-y1*x1-y2
 norid[113]=-y1*y2+x2^2*x1+x2*x1^2+y1
 norid[114]=-y1^2+y2*x2*x1
 norid[115]=z^3-y1^3*y2+y1^2*x2*x1^2-z*y1*x2*x1-y1^2*y2*x2-z*y1^2+y1^3-z*y1*x1-z*y2*x1
 norid[116]=-z^3*x2+y1^3*y2*x2-z^3*x1+z*y1^2*y2+y1^4+y1^2*y2*x2^2+z*y1^2*x2+z*y1^2*x1-y1^3*x
 norid[117]=-z^3*y2+y1^4*x2+z*y1^3+y1^3*x2^2+y1^3*x2*x1+z*y1^2*y2+z*y1*y2*x1+z*y1*x2*x1+z*y1
 norid[118]=y1^5-z^3*x2*x1-z^3*x1^2+z*y1^2*y2*x1+z^3*y2+y1^4*x1+z*y1^2*x2*x1+z*y1^2*x1^2-z*y

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> option(redSB);
> ideal j=std(norid);j;
j [1]=y2^2-y1*x2-y1*x1-y2
j [2]=y1*y2-x2^2*x1-x2*x1^2-y1
j [3]=y1^2-y2*x2*x1
j [4]=z^3-y2*x2^3*x1^2-z*y1*x2*x1-y1*x2^3*x1-y1*x2^2*x1^2-z*y2*x2*x1-z*y1*x1-z*y2*x1-y2*x2^2
j [5]=T(9)*z-y2*x2^4*x1-z*y1*x2^2-y1*x2^4-y1*x2^3*x1-z*y2*x2^2+x2^4*x1+x2^3*x1^2-z*y1*x2+y2*x2
j [6]=T(9)*y2*x1-z^2*y2*x2+z^2*x2+z^2*x1
j [7]=T(9)*y1*x1-z^2*y1*x2+z^2*y2-z^2
j [8]=T(9)*x2^2*x1+T(9)*x2*x1^2+T(9)*y1-z^2*x2^3-z^2*x2^2*x1+z^2*y1
j [9]=T(8)*x1-z*y1*x2+x2^2*x1^2+z*y2+y2*x2*x1+z*x1-z
j [10]=T(8)*y2-z*x2^3-z*x2^2*x1+y2*x2^2*x1+z*y1+y1*x2^2+y1*x2*x1+z*y2+y2*x2
j [11]=T(8)*y1-z*y2*x2^2+y1*x2^2*x1+z*x2^2+z*x2*x1+x2^3*x1+x2^2*x1^2+z*y1+y1*x2
j [12]=T(8)*z-T(9)*y1+z*x2^2*x1+z*y2*x2+z^2
j [13]=T(7)-T(9)*x2*x1^2+z^2*x2^2*x1-z^2*y1-z*x2^2*x1-z*y2*x2-z^2
j [14]=T(6)-y1*x2^2*x1-z*x2*x1-x2^3*x1-x2^2*x1^2-z*y1-z*x1-y1*x2
j [15]=T(5)-y2*x2^2*x1-z*y1-y1*x2^2-y1*x2*x1-z*y2+x2^2*x1+z
j [16]=T(4)+T(9)*x1-z^2*x2
j [17]=T(3)-x2^3*x1-x2^2*x1^2-z*y2-y2*x2^2-y2*x2*x1-z*x2-z*x1+x2^2*x1+y2*x2+z
j [18]=T(2)*x1-T(9)*x2*x1^2+z^2*x2^2*x1-z^2*y1
j [19]=T(2)*y2-T(2)+T(9)*x2*x1-z^2*x2^2
j [20]=T(2)*y1-z^2*x2
j [21]=T(2)*x2^2-T(9)*x2*x1^3-T(9)*y2+z^2*x2^2*x1^2-z^2*y1*x1+z^2*y2-2*z^2
j [22]=T(2)*z-y1*x2^3*x1-z*x2^2*x1-x2^4*x1-x2^3*x1^2-z*y1*x2-z*x2*x1-z*y1-y1*x2^2
j [23]=T(1)+x2^2*x1+y2*x2+z
j [24]=T(9)^2-T(8)*x2^4-T(8)*x2^2-T(9)*y1*x2^2-T(9)*y2*x2^2-T(9)*y1*x2+T(9)*x2^2+T(9)*x2*x1+
j [25]=T(8)*T(9)+T(8)*x2^2-T(8)-T(9)*x2*x1^2+T(9)*y2*x2-T(9)*y1-x2^7*x1-x2^6*x1^2-z*y2*x2^4-
j [26]=T(2)*T(9)+T(9)*x2*x1^2-T(9)*y1*x2-T(9)*x2*x1-z*y1*x2^4-z^2*x2^3-z^2*x2^2*x1-z*x2^5-z*
j [27]=T(8)^2-T(9)*y2*x2^2+T(9)*x2^2+T(9)*x2*x1+2*T(9)*y1+2*z*y1*x2^3+2*z*x2^4+2*z*x2^3*x1-x2
j [28]=T(2)*T(8)+T(2)*x2+T(9)*x2*x1^4+T(9)*x2*x1^2+T(9)*y1-z^2*x2^2*x1^3+z^2*y1*x1^2-y2*x2^5-
j [29]=T(2)^2-T(9)*x2*x1^4+T(9)*x2*x1^3+z^2*x2^2*x1^3-z^2*x2^2*x1^2-z^2*y1*x1^2-z*x2^4*x1+z^

```

The output of my char 0 code is

```

q= 3
GFq Finite field of size 3
Pq Polynomial ring of rank 5 over GF(3)
Order: Weight [full]
Variables: $.1, $.2, $.3, $.4, $.5
q 3
wt [
  [ 19, 15, 12, 9, 9 ],
  [ 12, 9, 9, 9, 0 ]
]
Q [
$.2^2 + 2*$.3*$.4*$.5,
$.2*$.3 + 2*$.4^2*$.5 + 2*$.4*$.5^2 + 2*$.2,

```



```

f_15_9*f_9_9^2*f_9_0 + f_15_9*f_9_9*f_9_0^2,
f_12_9*f_9_9^2*f_9_0 + f_12_9*f_9_9*f_9_0^2,
f_9_9^3*f_9_0 + f_9_9^2*f_9_0^2,
f_9_9^2*f_9_0^2 + f_9_9*f_9_0^3,
f_9_9^2*f_9_0 + f_9_9*f_9_0^2
]
1 f_19_12^2*f_9_9^2*f_9_0 + f_19_12^2*f_9_9*f_9_0^2
2 f_19_12^2*f_15_9*f_9_9 + f_19_12^2*f_15_9*f_9_0 - f_19_12^2*f_12_9 + f_19_12^23 f_19_12*f_
4 f_19_12^2*f_12_9*f_9_9 - f_19_12^2*f_9_9
5 f_19_12*f_12_9*f_9_9^2*f_9_0 + f_19_12*f_12_9*f_9_9*f_9_0^2
6 f_19_12*f_9_9^2*f_9_0 + f_19_12*f_9_9*f_9_0^2
7 f_15_9*f_9_9^2*f_9_0 + f_15_9*f_9_9*f_9_0^2
8 f_12_9*f_9_9^2*f_9_0 + f_12_9*f_9_9*f_9_0^2
9 f_9_9^3*f_9_0 + f_9_9^2*f_9_0^2
10 f_9_9^2*f_9_0^2 + f_9_9*f_9_0^3
11 f_9_9^2*f_9_0 + f_9_9*f_9_0^2
newrelations= [
f_12_9^2 - f_15_9*f_9_9 - f_15_9*f_9_0 - f_12_9,
f_15_9^2 - f_12_9*f_9_9*f_9_0,
f_15_9*f_12_9 - f_9_9^2*f_9_0 - f_9_9*f_9_0^2 - f_15_9,
f_19_12^2 - f_38_24,
f_19_12*f_15_9 - f_34_21,
f_19_12*f_12_9 - f_31_21,
f_31_21^2 - f_35_24*f_9_9^2*f_9_0 - f_35_24*f_9_9*f_9_0^2 -
2*f_32_24*f_9_9*f_9_0 - 2*f_32_24*f_9_0^2 - f_38_24,
f_31_21*f_19_12 - f_32_24*f_9_9*f_9_0 - f_32_24*f_9_0^2 - f_38_24,
f_31_21*f_15_9 - f_19_12*f_9_9^2*f_9_0 - f_19_12*f_9_9*f_9_0^2 - f_34_21,
f_31_21*f_12_9 - f_34_21*f_9_9 - f_34_21*f_9_0 - f_31_21,
f_32_24^2 - f_19_12*f_9_9^4*f_9_0 - f_32_24*f_9_9^2*f_9_0 - f_31_21*f_9_9^3
- f_38_24*f_9_9^2 - f_32_24*f_9_9*f_9_0 - f_38_24*f_9_9,
f_32_24*f_31_21 - f_9_9^5*f_9_0^2 - f_9_9^4*f_9_0^3 - f_31_21*f_9_9^2*f_9_0
- f_12_9*f_9_9^4*f_9_0 - f_12_9*f_9_9^3*f_9_0^2 - f_19_12*f_9_9^3*f_9_0
- f_19_12*f_9_9^2*f_9_0^2 - f_15_9*f_9_9^3*f_9_0 - f_31_21*f_9_9*f_9_0 -
f_9_9^3*f_9_0^2 - f_34_21*f_9_9 - f_34_21 - f_15_9*f_9_9^2,
f_32_24*f_19_12 - f_15_9*f_9_9^3*f_9_0 - f_19_12*f_9_9^2*f_9_0 -
f_9_9^4*f_9_0 - f_9_9^3*f_9_0^2 - f_34_21*f_9_9 - f_19_12*f_9_9*f_9_0 -
f_34_21 - f_15_9*f_9_9^2,
f_32_24*f_15_9 - f_38_24*f_9_9,
f_32_24*f_12_9 - f_35_24*f_9_9 - f_32_24,
f_34_21^2 - f_32_24*f_9_9^2*f_9_0^2 - f_32_24*f_9_9*f_9_0^3 -
f_38_24*f_9_9*f_9_0,
f_34_21*f_32_24 - f_12_9*f_9_9^4*f_9_0^2 - f_34_21*f_9_9^2*f_9_0 -
f_15_9*f_9_9^4*f_9_0 - f_15_9*f_9_9^3*f_9_0^2 - f_31_21*f_9_9^2*f_9_0 -
f_34_21*f_9_9*f_9_0 - f_31_21*f_9_9*f_9_0 - f_12_9*f_9_9^3*f_9_0,
f_34_21*f_31_21 - f_38_24*f_9_9^2*f_9_0 - f_38_24*f_9_9*f_9_0^2 -
f_35_24*f_9_9*f_9_0 - f_32_24*f_9_0,

```

$f_{34_21}f_{19_12} - f_{35_24}f_{9_9}f_{9_0} - f_{32_24}f_{9_0},$
 $f_{34_21}f_{15_9} - f_{31_21}f_{9_9}f_{9_0},$
 $f_{34_21}f_{12_9} - f_{19_12}f_{9_9}^2f_{9_0} - f_{19_12}f_{9_9}f_{9_0}^2 - f_{34_21},$
 $f_{35_24}^2 - f_{34_21}f_{9_9}^3f_{9_0} - f_{34_21}f_{9_9}^2f_{9_0}^2 -$
 $f_{38_24}f_{9_9}^2f_{9_0} - f_{38_24}f_{9_9}f_{9_0}^2 - f_{19_12}f_{9_9}^4f_{9_0} -$
 $2f_{19_12}f_{9_9}^3f_{9_0}^2 - f_{19_12}f_{9_9}^2f_{9_0}^3 -$
 $f_{35_24}f_{9_9}^2f_{9_0} - f_{35_24}f_{9_9}f_{9_0}^2 + f_{31_21}f_{9_9}^2f_{9_0} -$
 $f_{38_24}f_{9_9}f_{9_0} - f_{38_24}f_{9_0}^2 - f_{35_24}f_{9_0}^2 -$
 $f_{19_12}f_{9_9}^2f_{9_0} + f_{35_24}f_{9_0},$
 $f_{35_24}f_{34_21} - f_{15_9}f_{9_9}^4f_{9_0}^2 - f_{15_9}f_{9_9}^3f_{9_0}^3 -$
 $f_{19_12}f_{9_9}^3f_{9_0}^2 - f_{19_12}f_{9_9}^2f_{9_0}^3 - f_{9_9}^5f_{9_0}^2 -$
 $2f_{9_9}^4f_{9_0}^3 - f_{9_9}^3f_{9_0}^4 - f_{34_21}f_{9_9}^2f_{9_0} -$
 $f_{34_21}f_{9_9}f_{9_0}^2 - f_{19_12}f_{9_9}^2f_{9_0}^2 - f_{19_12}f_{9_9}f_{9_0}^3$
 $- f_{34_21}f_{9_9}f_{9_0} - f_{34_21}f_{9_0}^2 - f_{15_9}f_{9_9}^3f_{9_0} -$
 $f_{15_9}f_{9_9}^2f_{9_0}^2,$
 $f_{35_24}f_{32_24} - f_{31_21}f_{9_9}^3f_{9_0} - f_{35_24}f_{9_9}^2f_{9_0} -$
 $f_{34_21}f_{9_9}^3 - f_{34_21}f_{9_9}^2f_{9_0} - f_{32_24}f_{9_9}^2f_{9_0} -$
 $f_{32_24}f_{9_9}f_{9_0}^2 + f_{19_12}f_{9_9}^3f_{9_0} - f_{35_24}f_{9_9}f_{9_0} -$
 $f_{32_24}f_{9_9}f_{9_0} - f_{32_24}f_{9_0}^2,$
 $f_{35_24}f_{31_21} - f_{12_9}f_{9_9}^4f_{9_0}^2 - f_{12_9}f_{9_9}^3f_{9_0}^3 -$
 $f_{34_21}f_{9_9}^2f_{9_0} - f_{34_21}f_{9_9}f_{9_0}^2 - f_{15_9}f_{9_9}^4f_{9_0} -$
 $2f_{15_9}f_{9_9}^3f_{9_0}^2 - f_{15_9}f_{9_9}^2f_{9_0}^3 -$
 $f_{31_21}f_{9_9}^2f_{9_0} - f_{31_21}f_{9_9}f_{9_0}^2 - f_{34_21}f_{9_9}f_{9_0} -$
 $f_{34_21}f_{9_0}^2 - f_{31_21}f_{9_9}f_{9_0} - f_{31_21}f_{9_0}^2 -$
 $f_{12_9}f_{9_9}^3f_{9_0} - f_{12_9}f_{9_9}^2f_{9_0}^2,$
 $f_{35_24}f_{19_12} - f_{9_9}^4f_{9_0}^2 - f_{9_9}^3f_{9_0}^3 - f_{31_21}f_{9_9}f_{9_0} -$
 $f_{12_9}f_{9_9}^3f_{9_0} - f_{12_9}f_{9_9}^2f_{9_0}^2 - f_{19_12}f_{9_9}^2f_{9_0} -$
 $f_{19_12}f_{9_9}f_{9_0}^2 - f_{31_21}f_{9_0} - f_{19_12}f_{9_0}^2 + f_{19_12}f_{9_0},$ f_{35_24}
 $f_{35_24}f_{12_9} - f_{38_24}f_{9_9} - f_{38_24}f_{9_0},$
 $f_{38_24}^2 - f_{31_21}f_{9_9}^3f_{9_0}^2 - f_{35_24}f_{9_9}^2f_{9_0}^2 -$
 $f_{34_21}f_{9_9}^3f_{9_0} - f_{34_21}f_{9_9}^2f_{9_0}^2 -$
 $f_{32_24}f_{9_9}^2f_{9_0}^2 - f_{32_24}f_{9_9}f_{9_0}^3 - f_{35_24}f_{9_9}f_{9_0}^2$
 $- 2f_{32_24}f_{9_9}f_{9_0}^2 - f_{32_24}f_{9_0}^3 - f_{31_21}f_{9_9}^2f_{9_0} -$
 $f_{38_24}f_{9_9}f_{9_0} - f_{32_24}f_{9_0}^2 - f_{38_24}f_{9_0},$
 $f_{38_24}f_{35_24} - f_{19_12}f_{9_9}^4f_{9_0}^2 - f_{19_12}f_{9_9}^3f_{9_0}^3 -$
 $f_{32_24}f_{9_9}^2f_{9_0}^2 - f_{32_24}f_{9_9}f_{9_0}^3 - f_{31_21}f_{9_9}^3f_{9_0}$
 $- f_{31_21}f_{9_9}^2f_{9_0}^2 - f_{38_24}f_{9_9}^2f_{9_0} -$
 $f_{38_24}f_{9_9}f_{9_0}^2 - f_{32_24}f_{9_9}f_{9_0}^2 - f_{32_24}f_{9_0}^3 -$
 $f_{38_24}f_{9_9}f_{9_0} - f_{38_24}f_{9_0}^2,$
 $f_{38_24}f_{34_21} - f_{9_9}^5f_{9_0}^3 - f_{9_9}^4f_{9_0}^4 -$
 $f_{31_21}f_{9_9}^2f_{9_0}^2 - f_{12_9}f_{9_9}^4f_{9_0}^2 -$
 $f_{12_9}f_{9_9}^3f_{9_0}^3 - f_{19_12}f_{9_9}^3f_{9_0}^2 -$
 $f_{19_12}f_{9_9}^2f_{9_0}^3 - f_{15_9}f_{9_9}^3f_{9_0}^2 - f_{31_21}f_{9_9}f_{9_0}^2$ $- f_{9_9}^3f_{9_0}^3 - f_{34_21}f_{9_9}f_{9_0} - f_{34_21}f_{9_0} -$
 $f_{15_9}f_{9_9}^2f_{9_0},$
 $f_{38_24}f_{32_24} - f_{34_21}f_{9_9}^3f_{9_0} - f_{38_24}f_{9_9}^2f_{9_0} -$

```

f_19_12*f_9_9^4*f_9_0 - f_19_12*f_9_9^3*f_9_0^2 - f_35_24*f_9_9^2*f_9_0
- f_38_24*f_9_9*f_9_0 - f_35_24*f_9_9*f_9_0 - f_34_21*f_9_9^2 -
f_32_24*f_9_9*f_9_0 - f_32_24*f_9_0,
f_38_24*f_31_21 - f_15_9*f_9_9^4*f_9_0^2 - f_15_9*f_9_9^3*f_9_0^3 -
f_19_12*f_9_9^3*f_9_0^2 - f_19_12*f_9_9^2*f_9_0^3 - f_9_9^5*f_9_0^2 -
2*f_9_9^4*f_9_0^3 - f_9_9^3*f_9_0^4 - f_34_21*f_9_9^2*f_9_0 -
f_34_21*f_9_9*f_9_0^2 - f_12_9*f_9_9^3*f_9_0^2 - f_19_12*f_9_9^2*f_9_0^2 - f_31_21*f_9_9*f_9_0
- f_34_21*f_9_0 - f_31_21*f_9_0 - f_12_9*f_9_9^2*f_9_0,
f_38_24*f_19_12 - f_12_9*f_9_9^3*f_9_0^2 - f_34_21*f_9_9*f_9_0 -
f_15_9*f_9_9^3*f_9_0 - f_15_9*f_9_9^2*f_9_0^2 - f_31_21*f_9_9*f_9_0 -
f_34_21*f_9_0 - f_31_21*f_9_0 - f_12_9*f_9_9^2*f_9_0,
f_38_24*f_15_9 - f_35_24*f_9_9*f_9_0 - f_32_24*f_9_0,
f_38_24*f_12_9 - f_32_24*f_9_9*f_9_0 - f_32_24*f_9_0^2 - f_38_24
]
totaltime= 0.780 seconds

```

I can't get anything from MAGMA's Normalisation or any of the Macaulay2 1.2 commands at the moment, even in char 2, either for lack of space (4GB) or lack of time (20+hours CPU time).