Annotated Examples 5

Greuel, Laplagne, Seelich

I don't know the origin or reason to consider this, but it is example I_1 in Greuel, Laplagne, Seelisch, and is almost type I. (That is, it can be made type I by the simple change of variables $z = y - x$, $w = zx$.)

Well, OK, I still don't necessarily know the origin, but have found it in MAGMA's Normalisation description as an example with commentary:

```
> // now try a harder case - a singular affine form of modular curve X1(11)
> I := ideal<P | (x-y)*x*(y+x^2)^3-y^3*(x^3+x*y-y^2)>;
> time Js := Normalisation(I: FFMin := false);
Time: 0.110
> #Js;
1
> J := Js[1][1];
> Groebner(J);
> J;
Ideal of Polynomial ring of rank 5 over Rational Field
Lexicographical Order
Variables: $.1, $.2, $.3, $.4, $.5
Groebner basis:
[
    $.1*$.3 - $.1 - 6*$.3 + $.4*$.5^2 - 4*$.4*$.5 + 6*$.4 - $.5^5 + $.5^4 +
        11*$.5^3 - 16*$.5^2 + 2*$.5 + 6,
    $.1*$.4 + 2*$.3 - $.4*$.5^2 + 2*$.4*$.5 - 2*$.4 + $.5^4 - 4*$.5^3 + 4*$.5^2
        - 2,
    $.1*$.5 - 2*$.3 + $.4 + $.5^3 - 2*$.5^2 + $.5 + 1,
    $.2 - $.3 + $.5^3 - $.5^2,
    $.3^2 + 3*$.3 - 2*$.4*$.5^2 + 4*$.4*$.5 - 4*$.4 - $.5^6 + 2*$.5^5 + $.5^4 -
        10*$.5^3 + 10*$.5^2 - 4,
    $.3*$.4 - $.3 - $.4*$.5^3 + $.4*$.5^2 - $.4*$.5 + $.4 - $.5^4 + 2*$.5^3 -
        2*$.5^2 + 1,
    $.3*$.5 + $.3 - $.4 - $.5^4 + 2*$.5^2 - $.5 - 1,
    $.4^2 - 2*$.4*$.5^2 + $.4*$.5 + $.4 - $.5^5
]
> time Js := Normalisation(I);
Time: 1.110
> J := Js[1][1];
> Groebner(J);
> J;
Ideal of Polynomial ring of rank 2 over Rational Field
Lexicographical Order
Variables: $.1, $.2
Groebner basis:
[
```

```
    $.1^2*$.2 + 2*$.1*$.2 + $.1 - $.2^2 + 2*$.2 + 1
]
> // Minimised result is a cubic equation in K[x,y] - as good as we could get!
> // This example takes MUCH longer with the general method - even setting
> // UseMax := true.
```

It seems to be a curve of genus 1, so should have a presentation (as it does a few lines above) with *one* (I repeat *1*) relation in terms of functions of weights 3 and 2. Good luck figuring that out from anything in SINGULAR, Macaulay2, or MAGMA's IntegralClosure.

If this is run as given in SINGULAR 3-1-0 in characteristic 0,

```
                     SINGULAR                             /
 A Computer Algebra System for Polynomial Computations   /   version 3-1-0
                                                        0<
     by: G.-M. Greuel, G. Pfister, H. Schoenemann        \   Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern    \
> LIB "normal.lib";
> ring r=0,(x,y),dp;
> ideal i=(x-y)*x*(y+x^2)^3-y^3*(x^3+x*y-y^2);
> list nor=normal(i);
> nor;
[1]:
   [1]:
//   characteristic : 0
//   number of vars : 6
//        block   1 : ordering dp
//                  : names    T(1) T(2) T(3) T(4)
//        block   2 : ordering dp
//                  : names    x y
//        block   3 : ordering C
[2]:
   [1]:
      _[1]=x2y5
      _[2]=x4y4
      _[3]=x5y3+2x3y4+xy5
      _[4]=x7y2-3x3y4-2xy5
      _[5]=y6
> def R=nor[1][1];
> setring R;
> normap;
normap[1]=x
normap[2]=y
> norid;
norid[1]=2*T(1)*x+T(2)*x-T(3)*y+x
norid[2]=-T(1)*x+3*T(3)*x-3*T(3)*y+T(4)*x-T(4)*y-x+y
```

```
norid[3]=-T(1)*y+x^2
norid[4]=T(1)*x^2-T(2)*y
norid[5]=T(3)*x^2-2*T(3)*y-T(4)*y
norid[6]=T(1)^2-T(2)
norid[7]=T(1)*T(2)+T(1)+2*T(2)-T(3)*x
norid[8]=T(2)^2-3*T(1)*x-2*T(1)-T(2)*x-3*T(2)-T(3)*x^2+3*T(3)*x-2*x+y
norid[9]=T(1)*T(3)-2*T(3)-T(4)
norid[10]=T(2)*T(3)-3*T(1)*x-T(1)-T(2)*x-T(2)-T(3)*x^2+2*T(3)+T(4)-x+y
norid[11]=T(3)^2-3*T(1)*x-T(2)*x-T(2)-T(3)*x^2-4*T(3)-T(4)-x+y+1
norid[12]=T(1)*T(4)-3*T(1)*x-T(1)-T(2)*x-T(2)-T(3)*x^2+6*T(3)+3*T(4)-x+y
norid[13]=T(2)*T(4)-3*T(1)*x^3-T(1)*x^2+9*T(1)*x+4*T(1)-T(2)*x^3+2*T(2)*x+4*T(2)+3*T(3)*
norid[14]=T(3)*T(4)-3*T(1)*x^3-T(1)*x^2+3*T(1)*x+T(1)-T(2)*x^3+3*T(2)+T(3)*x^2-T(3)*x+6*
norid[15]=T(4)^2-T(1)*x^5+8*T(1)*x^3+3*T(1)*x^2-5*T(1)*x-6*T(1)+3*T(2)*x^3-T(2)*x^2+3*T(
norid[16]=x^8-x^7*y+3*x^6*y-3*x^5*y^2+3*x^4*y^2-4*x^3*y^3+x^2*y^3-2*x*y^4+y^5
> option(redSB);
> ideal j=std(norid);j;
j[1]=x^8-x^7*y+3*x^6*y-3*x^5*y^2+3*x^4*y^2-4*x^3*y^3+x^2*y^3-2*x*y^4+y^5
j[2]=6*T(4)*x^3-5*T(4)*x^2*y-T(4)*y^3+6*T(4)*x*y-3*T(4)*y^2-3*x^5-x^4-7*x^3*y+18*x^3+5*x
j[3]=T(4)*y^4-x^7+3*x^3*y^2+2*x*y^3
j[4]=T(4)*x*y^3-x^7+3*x^6-3*x^5*y+6*x^4*y-4*x^3*y^2+3*x^2*y^2-2*x*y^3+y^4
j[5]=T(4)*x^2*y^2+3*T(4)*y^3-x^7-3*x^5*y+6*x^5-x^4*y-4*x^3*y^2+12*x^3*y-x^2*y^2-x*y^3+y^
j[6]=6*T(3)*y^2+T(4)*x^2*y-T(4)*y^3+3*T(4)*y^2-3*x^5-x^4-7*x^3*y-x^2*y-3*x*y^2+y^3
j[7]=6*T(3)*x*y+T(4)*x^2*y-T(4)*y^3+2*T(4)*x*y+T(4)*y^2-3*x^5-x^4-7*x^3*y-2*x^3-x^2*y-3*
j[8]=T(3)*x^2-2*T(3)*y-T(4)*y
j[9]=2*T(2)*y-12*T(3)*y-T(4)*x^2*y+T(4)*y^3-2*T(4)*x^2-T(4)*y^2-6*T(4)*y+3*x^5+x^4+7*x^3
j[10]=T(2)*x+6*T(3)*x-7*T(3)*y+2*T(4)*x-2*T(4)*y-x+2*y
j[11]=T(1)*y-x^2
j[12]=T(1)*x-3*T(3)*x+3*T(3)*y-T(4)*x+T(4)*y+x-y
j[13]=3*T(4)^2-18*T(1)-30*T(2)-81*T(3)*x+180*T(3)*y-18*T(3)+4*T(4)*x^2*y-7*T(4)*y^3+15*T
j[14]=T(3)*T(4)+T(1)+3*T(2)+8*T(3)*x-13*T(3)*y+6*T(3)-T(4)*x^2+3*T(4)*x-5*T(4)*y+T(4)+x^
j[15]=T(2)*T(4)+4*T(1)+4*T(2)+14*T(3)*x-13*T(3)*y-6*T(3)-T(4)*x^2+5*T(4)*x-5*T(4)*y-3*T(
j[16]=T(1)*T(4)-T(1)-T(2)-3*T(3)*x+6*T(3)-T(4)*x+3*T(4)+x
j[17]=T(3)^2-T(2)-3*T(3)*x-4*T(3)-T(4)*x-T(4)+x+1
j[18]=T(2)*T(3)-T(1)-T(2)-3*T(3)*x+2*T(3)-T(4)*x+T(4)+x
j[19]=T(1)*T(3)-2*T(3)-T(4)
j[20]=T(2)^2-2*T(1)-3*T(2)-T(4)*x
j[21]=T(1)*T(2)+T(1)+2*T(2)-T(3)*x
j[22]=T(1)^2-T(2)
```

it is not too hard to figure out that $T(1) = x^2/y$, $T(2) = x^4/y^2$, $T(3) = x*(x^2+y)^2/y^3$ and $T(4)+3T(3) = x(x^2+y)^3/y^4$.

MAGMA gets a similar result if y is treated as the independent variable:

```
> FF<y>:=FunctionField(Q);
> P<x>:=PolynomialRing(FF);
> f:=(x-y)*x*(y+x^2)^3-y^3*(x^3+x*y-y^2);
> Ff<X>:=RationalExtensionRepresentation(FunctionField(f));
```

```
> C<Y>:=CoefficientRing(Ff);
> INT:=Integers(C);
> IC:=IntegralClosure(INT,Ff);
> B:=Basis(IC);for i in [1..#B] do i, B[i]; end for;
1 1
2 X
3 1/Y*X^2
4 1/Y*X^3
5 1/Y^2*X^4
6 1/Y^3*X^5 + 2/Y^2*X^3 + 1/Y*X
7 1/Y^3*X^6 + 3/Y^2*X^4 + 3/Y*X^2
8 1/Y^4*X^7 + 3/Y^3*X^5 + 3/Y^2*X^3 - 1/Y*X^2 + 1/Y*X
```

With the change of variables, MAGMA produces a predictable module basis of size 7, with implicit weights maybe 0,11,15,12,9,8,8, though these don't correspond to the implicit weights of the leading monomials

```
F:=Rationals();
P<y,x,w,z>:=PolynomialRing(F,4);
f1:=(x-y)*x*(y+x^2)^3-y^3*(x^3+x*y-y^2);1,f1;
f2:=y-x-z;
f3:=Resultant(f1,f2,y);
f4:=z*x-w;
f5:=Resultant(f3,f4,x) div z; f5;

FF<z>:=FunctionField(F);
P<w>:=PolynomialRing(FF);
f:=w^7 + 3*w^6*z + w^6 + 3*w^5*z^3 + 6*w^5*z^2 + 9*w^4*z^4 + 4*w^3*z^6 -
        w^3*z^5 - 3*w^2*z^7 - 3*w*z^9 - z^11;f;
Ff<W>:=RationalExtensionRepresentation(FunctionField(f));
C<z>:=CoefficientRing(Ff);
INT:=Integers(C);
IC:=IntegralClosure(INT,Ff);
B:=Basis(IC);
for i in [1..#B] do
i,B[i];
end for;
Genus(Ff);
Loading file "greuel1"
w^7 + (3*z + 1)*w^6 + (3*z^3 + 6*z^2)*w^5 + 9*z^4*w^4 + (4*z^6 - z^5)*w^3 -
    3*z^7*w^2 - 3*z^9*w - z^11
1 1
2 W
3 1/z*W^2 + 1/z*W
4 1/z^3*W^3 + (3*z + 1)/z^3*W^2 + 6/z*W
5 1/z^5*W^4 + (3*z + 1)/z^5*W^3 + (3*z + 6)/z^3*W^2 + 9/z*W
```

```
6 1/z^8*W^5 + (-2*z^2 + 3*z + 1)/z^8*W^4 + (3*z^2 - 3*z + 4)/z^6*W^3 + (-4*z +
    3)/z^3*W^2 + (-3*z^2 - 1)/z^3*W - 1/z
7 1/z^12*W^6 + (-z^3 - z^2 + 3*z + 1)/z^12*W^5 + (z^4 + 2*z^3 - 2*z^2 - z +
    5)/z^10*W^4 + (-3*z^4 + 3*z^2 - 4*z + 4)/z^8*W^3 + (6*z^5 - 2*z^4 + z^3 -
    1)/z^7*W^2 + (4*z^4 - z^3 + z - 2)/z^5*W + (-z^2 + z - 1)/z^3
1
```

My qth power algorithm gives a non-minimized ideal of relations:

```
f_2^2+30*f_4+30*f_3+5,
f_3^2+30*f_6+3*f_5+30*f_4+19*f_3+19*f_2+11,
f_3*f_2+30*f_5+f_4+3*f_3+17,
f_4^2+30*f_8+2*f_5+28*f_2,
f_4*f_3+30*f_7+3*f_6+6*f_5+29*f_4+19*f_3+13*f_2+30,
f_4*f_2+30*f_6+29*f_5+4*f_3+6*f_2+21,
f_5^2+30*f_3*f_7+29*f_8+4*f_6+21*f_5+6*f_4+11*f_3+14*f_2+29,
f_5*f_4+30*f_2*f_7+2*f_8+29*f_6+23*f_5+30*f_4+8*f_3+18*f_2+11,
f_5*f_3+30*f_8+3*f_7+28*f_6+16*f_5+27*f_4+18*f_3+2*f_2+4,
f_5*f_2+30*f_7+f_6+4*f_5+25*f_3+21*f_2+20,
f_6^2+30*f_5*f_7+3*f_4*f_7+26*f_3*f_7+16*f_2*f_7+8*f_8+f_7+29*f_6+12*f_5+15*f_4+7*f_
f_6*f_5+30*f_4*f_7+3*f_3*f_7+4*f_2*f_7+25*f_8+30*f_7+24*f_6+20*f_5+13*f_4+17*f_3+15*
f_6*f_4+30*f_3*f_7+29*f_8+f_7+9*f_6+22*f_5+2*f_4+26*f_3+2*f_2+28,
f_6*f_3+30*f_2*f_7+5*f_8+28*f_7+22*f_6+8*f_5+f_4+19*f_3+23*f_2+28,
f_6*f_2+30*f_8+f_7+f_6+27*f_5+f_4+6*f_3+9*f_2+26,
f_8^2+30*f_2*f_7^2+2*f_8*f_7+28*f_6*f_7+27*f_5*f_7+30*f_4*f_7+6*f_3*f_7+2*f_2*f_7+5*
f_8*f_6+30*f_7^2+3*f_6*f_7+4*f_5*f_7+28*f_4*f_7+3*f_3*f_7+14*f_2*f_7+2*f_8+11*f_7+23
f_8*f_5+30*f_6*f_7+27*f_2*f_7+3*f_8+3*f_7+3*f_6+12*f_5+6*f_4+17*f_3+29*f_2+9,
f_8*f_4+30*f_5*f_7+f_2*f_7+2*f_8+30*f_6+25*f_5+30*f_4+4*f_3+12*f_2+21,
f_8*f_3+30*f_4*f_7+3*f_3*f_7+6*f_2*f_7+21*f_8+22*f_7+28*f_6+26*f_5+26*f_4+15*f_3+24*
f_8*f_2+30*f_3*f_7+29*f_2*f_7+2*f_8+3*f_7+f_6+2*f_5+f_4+26*f_3+23*f_2+15
```

```
time1= 10.100
```

Minimizing first produces

```
f_15-f_3*f_2^6-12*f_3*f_2^5-2*f_2^6-51*f_3*f_2^4+13*f_2^5-104*f_3*f_2^3+129*f_2^4-108*f_
f_14-f_2^7+2*f_3*f_2^5-6*f_2^6+15*f_3*f_2^4-20*f_2^5+44*f_3*f_2^3-68*f_2^4+63*f_3*f_2^2
f_13-f_3*f_2^5+f_2^6-13*f_3*f_2^4-f_2^5-40*f_3*f_2^3+29*f_2^4-52*f_3*f_2^2+131*f_2^3-40
f_12-f_2^6+3*f_3*f_2^4-3*f_2^5+16*f_3*f_2^3-13*f_2^4+34*f_3*f_2^2-59*f_2^3+35*f_3*f_2-1
f_11-f_3*f_2^4-f_2^5-7*f_3*f_2^3-3*f_2^4-18*f_3*f_2^2+11*f_2^3-20*f_3*f_2+57*f_2^2-8*f_
f_10-f_2^5+4*f_3*f_2^3+f_2^4+11*f_3*f_2^2-13*f_2^3+5*f_3*f_2-59*f_2^2-2*f_3-40*f_2+4,
 f_9-f_3*f_2^3-2*f_2^4-3*f_3*f_2^2-2*f_2^3-2*f_3*f_2+8*f_2^2+8*f_2,
 f_8-f_2^4+2*f_3*f_2^2-f_2^3+4*f_3*f_2-8*f_2^2+f_3-20*f_2-5,
 f_7-f_3*f_2^2-2*f_2^3-3*f_3*f_2-2*f_2^2-2*f_3+8*f_2+8,
 f_3^2-f_2^3+6*f_3*f_2+4*f_2^2-5*f_3-23*f_2+2,
 f_6-f_2^3+3*f_3*f_2+2*f_2^2-11*f_2-8,
 f_5-f_3*f_2-f_2^2-2*f_3+9,
 f_4-f_2^2+f_3-5
```

from which

```
f_3^2-f_2^3+6*f_3*f_2+4*f_2^2-5*f_3-23*f_2+2
```

can be read off. After all a genus computation gives $g = 1$.

SINGULAR is very quick, as claimed, but produces the following $7 + 2$ variable 42 relation answer. Good luck reading any of it, let alone recovering the elliptic curve from it. I guess "speed kills".

```
> LIB "normal.lib";
> ring r=0,(w,z),wp(11,7);
> ideal i=w^7+3*w^6*z+w^6+3*w^5*z^3+6*w^5*z^2+9*w^4*z^4+4*w^3*z^6-w^3*z^5-3*w^2*z^7-3*w*
> list nor=normal(i);
> nor;
[1]:[1]:
//   characteristic : 0
//   number of vars : 9
//        block   1 : ordering dp
//                  : names    T(1) T(2) T(3) T(4) T(5) T(6) T(7)
//        block   2 : ordering wp
//                  : names    w z
//                  : weights  11 7
//        block   3 : ordering C
[2]:
   [1]:
      _[1]=-72wz10+324w6z2+288w4z5-216w2z8+311w7+550w5z3-216w3z6+262w6z-59w4z4+26w5z2+13
      _[2]=972w7z-396wz10+1188w6z2+1584w4z5-1188w2z8+1121w7+3034w5z3-1188w3z6+1450w6z-32
      _[3]=2187w8+117wz10-351w6z2-468w4z5+351w2z8-331w7-896w5z3+351w3z6-428w6z+97w4z4-40
      _[4]=78732w2z10-236196w7z2-314928w5z5+236196w3z8-236196w8-708588w6z3+236196w4z6-47
      _[5]=36wz10-108w6z2-108w4z5+108w2z8-103w7-206w5z3+108w3z6-98w6z+31w4z4-10w5z2-5w6
      _[6]=162w5z4+198wz10-594w6z2-792w4z5+594w2z8-721w7-1514w5z3+594w3z6-722w6z+163w4z4
      _[7]=-9wz10+27w6z2+36w4z5-18w2z8+4w7+26w5z3-27w3z6-10w6z-40w4z4-26w5z2-4w6
      _[8]=3w7+6w5z3+4w3z6+6w6z+9w4z4+6w5z2+w6
> def R=nor[1][1];
> setring R;
> normap;
normap[1]=w
normap[2]=z
> norid;
norid[1]=-324*T(1)*z-126*T(1)-T(2)-8*T(3)-648*T(5)*z+27*T(5)-48*T(6)+5832*w
norid[2]=52488*T(1)*w+47628*T(1)*z+12330*T(1)-5832*T(2)*z+4123*T(2)-1768*T(3)+48*T(4)-63
norid[3]=257580*T(1)*z+399510*T(1)+52488*T(2)*w+25309*T(2)-23328*T(3)*z+11336*T(3)+264*T
norid[4]=-38151*T(1)*z+560880*T(1)+1181*T(2)+26244*T(3)*w+37684*T(3)-729*T(4)*z-39*T(4)-
norid[5]=-57348*T(1)*z-35082*T(1)-2291*T(2)-952*T(3)-24*T(4)+52488*T(5)*w+9369*T(5)-1166
norid[6]=-66582*T(1)*z-36252*T(1)-729*T(2)*z-1294*T(2)-1664*T(3)-12*T(4)-59049*T(5)*z+86
norid[7]=78732*T(1)*z^2+254178*T(1)*z+109602*T(1)-7695*T(2)*z+11939*T(2)+2592*T(3)*z-56*
norid[8]=35964*T(1)*z+15858*T(1)-108*T(2)*z+629*T(2)+108*T(3)*z+688*T(3)+6*T(4)+42282*T(
```

```
norid[9]=-634567770*T(1)*z-554409162*T(1)+38263752*T(2)*z^3+65938050*T(2)*z^2+20180421*T
norid[10]=576301230*T(1)*z+491351652*T(1)-26572050*T(2)*z^2-29333259*T(2)*z+42232058*T(2
norid[11]=-3858717401154*T(1)*z-843314560434*T(1)-41776143984*T(2)*z^2+95793909903*T(2)*
norid[12]=-941209264908*T(1)*z-952032991500*T(1)+18255588840*T(2)*z^2+77886428769*T(2)*z
norid[13]=-50200592301626310*T(1)*z-47348968075492896*T(1)+811902928850208*T(2)*z^2+3847
norid[14]=T(1)^2-1256160960/479233*T(1)*w*z+239774763876480/229664268289*T(1)*z^2-186723
norid[15]=T(1)*T(2)-5426740800/479233*T(1)*w*z+822422696188896/229664268289*T(1)*z^2-805
norid[16]=T(2)^2-21695279760/479233*T(1)*w*z+1793463050814336/229664268289*T(1)*z^2-3213
norid[17]=T(1)*T(3)-5498642880/479233*T(1)*w^2*z+1841377965603264/229664268289*T(1)*w*z^
norid[18]=T(2)*T(3)-30242535840/479233*T(1)*w^2*z+10127578810817952/229664268289*T(1)*w*
norid[19]=T(3)^2+17870589360/479233*T(1)*w^2*z-5984478388210608/229664268289*T(1)*w*z^2+
norid[20]=T(1)*T(4)+512639447579025984/229664268289*T(1)*w^2*z-126729546624/479233*T(1)*
norid[21]=T(2)*T(4)+2042812063088713872/229664268289*T(1)*w^2*z-697012506432/479233*T(1)
norid[22]=T(3)*T(4)-2418105484385777880/229664268289*T(1)*w^3*z-188444042624144720247134
norid[23]=T(4)^2-423699231468985357104/229664268289*T(1)*w^3*z+138578759233344/479233*T(
norid[24]=T(1)*T(5)+496082880/479233*T(1)*w*z-34029213479136/229664268289*T(1)*z^2+73647
norid[25]=T(2)*T(5)+2769664752/479233*T(1)*w*z+71858079187776/229664268289*T(1)*z^2+4064
norid[26]=T(3)*T(5)+2749321440/479233*T(1)*w^2*z-920688982801632/229664268289*T(1)*w*z^2
norid[27]=T(4)*T(5)-187147531802598192/229664268289*T(1)*w^2*z+63364773312/479233*T(1)*z
norid[28]=T(5)^2-136786320/479233*T(1)*w*z+585142009344/229664268289*T(1)*z^2-2020789770
norid[29]=T(1)*T(6)+3508046496/479233*T(1)*w*z-596462009290560/229664268289*T(1)*z^2+103
norid[30]=T(2)*T(6)+14185315836/479233*T(1)*w*z-2443986353076192/229664268289*T(1)*z^2+4
norid[31]=T(3)*T(6)+15121267920/479233*T(1)*w^2*z-5063789405408976/229664268289*T(1)*w*z
norid[32]=T(4)*T(6)-1232875304190067896/229664268289*T(1)*w^2*z+348506253216/479233*T(1)
norid[33]=T(5)*T(6)-1460261520/479233*T(1)*w*z+54149889799296/229664268289*T(1)*z^2-4317
norid[34]=T(6)^2+586710864/479233*T(1)*z^3-7581237606/479233*T(1)*w*z+5515682136388866/2
norid[35]=T(1)*T(7)-35393760/479233*T(1)*w*z-17700721634952/229664268289*T(1)*z^2-207018
norid[36]=T(2)*T(7)-593282880/479233*T(1)*w*z-550283087576664/229664268289*T(1)*z^2-3460
norid[37]=T(3)*T(7)-687330360/479233*T(1)*w^2*z+230172245700408/229664268289*T(1)*w*z^2-
norid[38]=T(4)*T(7)+18129831984641988/229664268289*T(1)*w^2*z-15841193328/479233*T(1)*z^
norid[39]=T(5)*T(7)+18168300/479233*T(1)*w*z+8692492336200/229664268289*T(1)*z^2+1063256
norid[40]=T(6)*T(7)-4258170/479233*T(1)*w*z+49720772197152/229664268289*T(1)*z^2-3760511
norid[41]=T(7)^2+2293515/479233*T(1)*w*z-5330028431340/229664268289*T(1)*z^2+142545405/1
norid[42]=-w^7+z^11-3*w^5*z^3-4*w^3*z^6+3*w*z^9-3*w^6*z-9*w^4*z^4+3*w^2*z^7-6*w^5*z^2+w^
```

I can't get Macaulay2's integralClosure to work on this, so I pared it down to char 5 and still only got icFracP to work. I reduced the 7 fractions to get things of weights 9,3,6,5,7,0,2, of which of course only the 3 and 2 are necessary.

```
Macaulay 2, version 1.2
i1 : R=ZZ/5[w,z,MonomialOrder=>{Weights=>{11,7}}]
i2 : I=ideal(w^7+3*w^6*z+w^6+3*w^5*z^3+6*w^5*z^2+9*w^4*z^4
            +4*w^3*z^6-w^3*z^5-3*w^2*z^7-3*w*z^9-z^11)

i3 : S=R/I
i5 : F=icFracP(S)
```

```
       5     3 4     4 2    2 5    5    3 3      6    4     2 4    7     3 2
      w z - w z  - 2w z  + w z  + w  + w z  + w*z  + w z + w z  + z  + 2w z  - 2o5 = {---
                                                         w z - w z  + z  - 2w z  + w  + 2z  + 2w     ---
     w*z  + 2w  - w z  - 2z  - 2w z - w*z  + w z  - w  - w*z  - w z
     -------------------------------------------------------------,
     2 2     5     3      3    2
      z  - 2z  + 2w  - w*z  + w z
     ---------------------------------------------------------------------------     2 2
     w z  + 2z  - w  + 2w*z   z  - w z + 2w*z  + 2w z  + w
     ----------------------, -----------------------------,
         3      3    2                   2 2    3
        w  + w*z  + w z                 w z  + w
     ---------------------------------------------------------------------------      5
     w*z  - w  + z  - 2w z - w z  + z  - w  + w*z
     ---------------------------------------------,
               4    2 2    5     3      3
            w*z  - w z  + z  + 2w  + w*z
     ---------------------------------------------------------------------------      6
     w*z  - w z + 2w z  + w  - 2w z  + z  + 2w z - 2w z  + 2z  + 2w  + 2w*z
     ----------------------------------------------------------------------, 1,
                          w z  + w z + z  - w  + w*z
     ---------------------------------------------------------------------------      5
      z  - 2w  - w z
     ---------------}
      3      3    2
     w  + w*z  + w z

i7 : f0=F#0-w

        5    3 3       6     2 4     7     3 2       5     4    2 3     3
     - w  + w z  - 2w*z  - 2w z  - 2z  + 2w z  - 2w*z  - 2w  - w z  - 2w z - 2w*o7 = ---
                               w*z  + w z  + 2z  + w z + 2w*z  + 2z  + 2w  + w*z
     ---------------------------------------------------------------------------     4
     z  - w z  + 2w  + w*z  + w z
     ---------------------------
        2
     - w z

i8 : f1=F#1

      2 2     5    3       3
     w z  + 2z  - w  + 2w*z
o8 = ----------------------
         3      3    2
        w  + w*z  + w z
```

```
i9 : f2=F#2

       6    3        4     2 2   3
      z  - w z + 2w*z  + 2w z  + w
o9 = ----------------------------
               2 2   3
              w z  + w

i11 : f3=F#3-z

         4    2 3    3       4    2 2    5    3      3
       - w  + w z  + w z - w*z  - w z  + z  - w  + w*z
o11 = -----------------------------------------------
                   4    2 2    5     3      3
                w*z  - w z  + z  + 2w  + w*z

i12 : f4=F#4

         6    4      2 4    4     2 3    6     3      2 2     5     3       3
       w*z  - w z + 2w z  + w  - 2w z  + z  + 2w z - 2w z  + 2z  + 2w  + 2w*z
o12 = -----------------------------------------------------------------------
                                2 3    3     5    3      3
                               w z  + w z + z  - w  + w*z

i13 : f5=F#5

o13 = 1

i14 : f6=F#6

        5     3    2
        z  - 2w  - w z
o14 = ---------------
        3      3    2
       w  + w*z  + w z
```

And normalP with the new "withRing" and "noRed" options gives

```
                    SINGULAR                             /
 A Computer Algebra System for Polynomial Computations   /   version 3-1-0
                                                       0<
     by: G.-M. Greuel, G. Pfister, H. Schoenemann        \   Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern    \
> LIB "/home/leonada/presolve.lib";
```

```
> LIB "/home/leonada/normal.lib";
> ring r=5,(w,z),wp(11,7);
> ideal i=w^7+3*w^6*z+w^6+3*w^5*z^3+6*w^5*z^2+9*w^4*z^4+4*w^3*z^6-w^3*z^5-3*w^2*z^7-3*w*
> list norp=normalP(i,"withRing","noRed");

// 'normalP' created a list, say nor, of three lists:
// nor[1] resp. nor[2] are lists of 1 ring(s) resp. ideal(s)
// and nor[3] is a list of an intvec and an integer.
// To see the result, type
     nor;
// To access the i-th ring nor[1][i] give it a name, say Ri, and type e.g.
     def R1 = nor[1][1]; setring R1;  norid; normap;
// for the other rings type first setring r; (if r is the name of your
// original basering) and then continue as for the first ring;
// Ri/norid is the affine algebra of the normalization of the i-th
// component r/P_i (where P_i is an associated prime of the input ideal)
// and normap the normalization map from r to Ri/norid;
//    nor[2] is a list of 1 ideal(s), each ideal nor[2][i] consists of
// elements g1..gk of r such that the gj/gk generate the integral
// closure of r/P_i as (r/P_i)-module in the quotient field of r/P_i.
//    nor[3] shows the delta-invariant of each component and of the input
// ideal (-1 means infinite, and 0 that r/P_i is normal).
> norp;
[1]:
   [1]:
      //   characteristic : 5
//   number of vars : 8
//        block   1 : ordering dp
//                  : names    T(1) T(2) T(3) T(4) T(5) T(6)
//        block   2 : ordering wp
//                  : names    w z
//                  : weights  11 7
//        block   3 : ordering C
[2]:
   [1]:
      _[1]=w2z6-w3z4-wz7+2w2z5+2w5+2w3z3-w4z
      _[2]=w4z3+2z9+w3z4+wz7-w4z2+2w2z5-2w5-w3z3+w4z
      _[3]=w6+2z9+w3z4+2w4z2-2w2z5+w5-w3z3-w4z
      _[4]=wz8-z9-w3z4+wz7+2w4z2-w2z5+2w5+w3z3-w4z
      _[5]=w3z5-z9+2w3z4+2wz7+2w4z2+w2z5+2w3z3-w4z
      _[6]=z10+2z9+wz7+w2z5+w5+2w3z3
      _[7]=w5z+w3z4+2wz7-w5-w3z3+w4z
[3]:
   [1]:
      29
   [2]:
```

```
      29
> def R=norp[1][1];
> setring R;
> normap;
normap[1]=w
normap[2]=z
> norid;
norid[1]=-2*T(1)*z-T(1)-T(2)*z+2*T(2)-T(3)*z+T(3)-T(4)-T(6)+w+2
norid[2]=T(1)*w+T(1)*z+2*T(1)+T(2)*z+T(2)-2*T(3)+2*T(4)-T(5)*z+2*T(6)+1
norid[3]=-T(1)*w+2*T(1)*z-T(1)-2*T(2)*w+T(2)*z+2*T(2)+T(3)*w+T(3)-T(4)*z-T(4)-T(6)*z-T(6
norid[4]=-T(1)*w-T(2)*w+z^2
norid[5]=T(1)*z^2+T(1)*w-T(4)*w
norid[6]=-T(1)*w-T(1)*z-2*T(1)+T(2)*z^2+2*T(2)*z-T(2)+2*T(3)-2*T(4)*z-2*T(4)-T(5)*w-2*T(
norid[7]=T(1)*w-2*T(1)*z+T(1)-T(2)*z-2*T(2)-T(3)+T(4)*z^2+T(4)*z+T(4)-T(6)*w+T(6)*z+T(6)
norid[8]=T(1)^2-2*T(2)-T(3)
norid[9]=T(1)*T(2)+T(1)+2*T(2)+T(3)-T(4)
norid[10]=T(2)^2+2*T(2)+2*T(3)+T(4)-T(5)-1
norid[11]=T(1)*T(3)+2*T(1)-T(2)+T(3)+T(4)-T(5)-2
norid[12]=T(2)*T(3)-T(1)+2*T(3)+T(4)+2*T(5)-z
norid[13]=T(3)^2-2*T(1)+T(2)-2*T(3)+2*T(4)+T(5)-T(6)-z+2
norid[14]=T(1)*T(4)-2*T(1)-2*T(3)+2*T(4)-T(5)-z+1
norid[15]=T(2)*T(4)+2*T(1)-2*T(2)-2*T(3)-2*T(4)+T(5)-T(6)+z+2
norid[16]=T(3)*T(4)-T(1)*z+2*T(1)+2*T(2)+T(3)+T(5)+T(6)+z+1
norid[17]=T(4)^2-2*T(1)*z+2*T(1)-T(2)*z+T(2)-2*T(3)-2*T(4)+2*T(6)+2*z+1
norid[18]=T(1)*T(5)+2*T(1)-T(2)-2*T(3)-2*T(4)-2*T(5)-T(6)+z+1
norid[19]=T(2)*T(5)-T(1)*z+2*T(1)-T(2)-2*T(3)+T(4)+T(5)+T(6)-z-1
norid[20]=T(3)*T(5)+T(1)*z-2*T(1)-T(2)*z-2*T(4)-2*T(6)
norid[21]=T(4)*T(5)+2*T(1)*z-2*T(1)-T(2)*z-T(2)-2*T(3)+T(4)+T(5)-T(6)-w+z-2
norid[22]=T(5)^2+2*T(1)*z-T(1)-2*T(2)*z-T(4)*z-T(4)+T(5)+T(6)+w-2*z-2
norid[23]=T(1)*T(6)-2*T(1)*z+T(1)-T(2)*z+T(2)-T(3)-T(4)+2*T(6)-2*z-2
norid[24]=T(2)*T(6)-T(2)*z-T(2)-2*T(3)-T(5)-w-z-2
norid[25]=T(3)*T(6)+T(1)-2*T(2)*z+2*T(2)-T(3)-T(4)*z+2*T(4)+2*T(6)+w
norid[26]=T(4)*T(6)-T(1)*w+T(1)*z+T(1)+T(2)*z+2*T(2)+T(3)-2*T(4)*z-T(4)-T(5)+2*T(6)+2*z-
norid[27]=T(5)*T(6)-T(1)*w-T(1)-2*T(2)*z+T(2)-2*T(3)+2*T(4)*z-2*T(4)+T(5)-T(6)-z^2+2*w+z
norid[28]=T(6)^2-T(1)*z^2-T(1)*w+2*T(1)*z+2*T(1)-2*T(2)*z+T(2)-2*T(3)*w+2*T(3)+2*T(4)*z+
norid[29]=w^7-z^11-2*w^5*z^3-w^3*z^6+2*w*z^9-2*w^6*z-w^4*z^4+2*w^2*z^7+w^5*z^2-w^3*z^5+w
> option(redSB);
> ideal j=std(norid);j;
j[1]=w^7-z^11-2*w^5*z^3-w^3*z^6+2*w*z^9-2*w^6*z-w^4*z^4+2*w^2*z^7+w^5*z^2-w^3*z^5+w^6
j[2]=T(6)*w^2*z^2+2*T(6)*w^3+T(6)*w*z^3+T(6)*w^2*z-w^4-2*w^2*z^3+w^3*z+2*w*z^4+w^2*z^2-2
j[3]=T(6)*w*z^5-T(6)*z^6+T(6)*w^3*z-2*T(6)*w*z^4-T(6)*w^3-2*T(6)*w*z^3-2*T(6)*w^2*z-w^3*
j[4]=T(6)*z^7-T(6)*w^4-T(6)*z^6-T(6)*w^3*z-T(6)*w*z^4+2*T(6)*w^3+2*T(6)*w*z^3+2*T(6)*w^2
j[5]=T(6)*w^5-2*T(6)*w^4*z-2*T(6)*z^6-2*T(6)*w^3*z+2*T(6)*w*z^3+2*T(6)*w^2*z-z^9+w^5*z-w
j[6]=T(5)*z^3+T(5)*w*z-2*T(6)*w*z^3+T(6)*w^2*z+2*T(6)*z^4+2*T(6)*w*z^2+2*T(6)*w^2+2*T(6)
j[7]=T(5)*w*z^2+T(5)*w^2-T(6)*w*z^3-2*T(6)*w^2*z+T(6)*w*z^2+T(6)*w^2+w^3*z+2*w*z^4-2*w^2
j[8]=T(5)*w^2*z+2*T(6)*w*z^4-2*T(6)*z^5+2*T(6)*w^3+2*T(6)*w*z^3-T(6)*z^4+2*T(6)*w*z^2-2*
```

```
j[9]=T(5)*w^3+T(6)*z^5+2*T(6)*w*z^3+T(6)*w^2*z-w^2*z^3-2*z^6+w*z^4+2*w^2*z^2-2*z^5-2*w*z
j[10]=T(4)*w-T(5)*w^2+2*T(5)*w*z+2*T(5)*z^2+2*T(5)*w-2*T(6)*w*z^3+T(6)*w^2*z+2*T(6)*z^4-
j[11]=T(4)*z^2-2*T(5)*z^2-2*T(5)*w-T(6)*w*z^2-2*T(6)*w^2+T(6)*z^3-2*T(6)*w*z-T(6)*z^2-T(
j[12]=T(3)*w+T(5)*w^2-2*T(5)*w*z-T(5)*z^2+2*T(6)*w*z^3-T(6)*w^2*z-2*T(6)*z^4-T(6)*w*z^2-
j[13]=T(3)*z^2+T(5)*z^2-2*T(6)*w*z^2+T(6)*w^2+2*T(6)*z^3+T(6)*w*z-T(6)*w+2*w^3-w*z^3-w^2
j[14]=T(2)*z+T(4)*z-T(5)*w^2+2*T(5)*w*z-T(5)*w-2*T(5)*z-2*T(6)*w*z^3+T(6)*w^2*z+2*T(6)*z
j[15]=T(2)*w-2*T(5)*w^2-T(5)*w*z+T(5)*z^2-T(5)*w+T(6)*w*z^3+2*T(6)*w^2*z-T(6)*z^4-T(6)*w
j[16]=T(1)-2*T(2)-2*T(3)*z-T(3)-2*T(4)*z+T(4)-T(5)*w^2+2*T(5)*w*z-T(5)*z^2-2*T(5)*w-2*T(
j[17]=T(6)^2+T(3)*z-T(3)+2*T(4)*z-T(5)*w^2+2*T(5)*w*z+T(5)*z^2+T(5)*w-T(5)*z-T(5)-2*T(6)
j[18]=T(5)*T(6)-T(2)-2*T(3)*z+2*T(3)+2*T(4)*z-T(4)-T(5)*w^2+2*T(5)*w*z-2*T(5)*z^2+2*T(5)
j[19]=T(4)*T(6)-T(2)-2*T(3)*z+2*T(3)+T(4)*z-2*T(4)-2*T(5)*w^2-T(5)*w*z+2*T(5)*z^2+T(5)*z
j[20]=T(3)*T(6)-T(2)+2*T(3)*z-2*T(4)*z+T(4)-T(5)*w^2+2*T(5)*w*z+T(5)*z^2+T(5)*z-2*T(6)*w
j[21]=T(2)*T(6)-T(2)-2*T(3)+T(4)*z-T(5)*w^2+2*T(5)*w*z-T(5)*w-2*T(5)*z-T(5)-2*T(6)*w*z^3
j[22]=T(5)^2-2*T(2)-T(3)-2*T(4)*z-2*T(5)*z^2-2*T(5)*w-T(5)*z+T(5)-T(6)*w*z^2-2*T(6)*w^2+
j[23]=T(4)*T(5)-2*T(3)*z+T(3)+T(4)*z-2*T(4)+2*T(5)*z^2+2*T(5)*w+T(5)*z+T(5)+T(6)*w*z^2+2
j[24]=T(3)*T(5)+T(2)+2*T(3)*z-2*T(3)-T(4)*z+T(5)*w^2-2*T(5)*w*z+T(5)*w+2*T(5)*z+2*T(6)*w
j[25]=T(2)*T(5)-2*T(2)-2*T(3)*z+2*T(4)*z-T(4)-2*T(5)*w^2-T(5)*w*z-2*T(5)*w+T(5)*z+T(5)+T
j[26]=T(4)^2+2*T(3)*z+T(4)*z+T(4)-2*T(5)*w^2-T(5)*w*z-2*T(5)*z^2+T(5)*w+T(6)*w*z^3+2*T(6
j[27]=T(3)*T(4)+T(2)-2*T(3)*z-2*T(3)+2*T(4)*z-2*T(4)-2*T(5)*w^2-T(5)*w*z-2*T(5)*w+T(5)*z
j[28]=T(2)*T(4)+2*T(2)-T(3)*z-T(4)*z+T(4)+2*T(5)*w^2+T(5)*w*z+2*T(5)*z^2-T(5)*w+T(5)-T(6
j[29]=T(3)^2+2*T(2)+T(3)*z+T(3)+T(4)*z-T(4)-2*T(5)*w^2-T(5)*w*z-2*T(5)*z^2+T(5)*w+T(5)+T
j[30]=T(2)*T(3)-2*T(2)-2*T(3)*z+T(3)-2*T(4)*z+2*T(4)-T(5)*w^2+2*T(5)*w*z-T(5)*z^2-2*T(5)
j[31]=T(2)^2+2*T(2)+2*T(3)+T(4)-T(5)-1
```

again having no clue that this is a disguised elliptic curve example, whereas without the “noRed” option, the same set of fractions gives rise to a supposedly minimal presentation which is still not minimal enough:

```
list norp=normalP(i,"withRing");
> norp;
[1]:
   [1]:
//   characteristic : 5
//   number of vars : 4
//        block   1 : ordering dp
//                  : names    T(2) T(5) T(6)
//        block   2 : ordering wp
//                  : names    z
//                  : weights  7
//        block   3 : ordering C
[2]:
   [1]:
      _[1]=w2z6-w3z4-wz7+2w2z5+2w5+2w3z3-w4z
      _[2]=w4z3+2z9+w3z4+wz7-w4z2+2w2z5-2w5-w3z3+w4z
      _[3]=w6+2z9+w3z4+2w4z2-2w2z5+w5-w3z3-w4z
      _[4]=wz8-z9-w3z4+wz7+2w4z2-w2z5+2w5+w3z3-w4z
```

```
      _[5]=w3z5-z9+2w3z4+2wz7+2w4z2+w2z5+2w3z3-w4z
      _[6]=z10+2z9+wz7+w2z5+w5+2w3z3
      _[7]=w5z+w3z4+2wz7-w5-w3z3+w4z
[3]:
   [1]:
      29
   [2]:
      29
> def R=norp[1][1];
> setring R;
> normap;
normap[1]=-T(2)^3-2*T(2)^2+T(2)*T(5)+T(2)*T(6)-T(2)*z-2*T(2)-T(5)-T(6)-2*z
normap[2]=z
> norid;
norid[1]=T(2)^4*z-2*T(2)^4-2*T(2)^3-T(2)^2*T(5)*z+2*T(2)^2*T(5)+2*T(2)*T(5)*z-2*T(2)*T(5
norid[2]=2*T(2)^7-2*T(2)^6+T(2)^5*T(5)-2*T(2)^5*T(6)+2*T(2)^5*z-T(2)^4*T(5)+2*T(2)^3*T(5
norid[3]=2*T(2)^6-2*T(2)^5+T(2)^4*T(5)-2*T(2)^4*T(6)-T(2)^4*z-2*T(2)^4-T(2)^3*T(5)+2*T(2
norid[4]=2*T(2)^4*z-2*T(2)^4-T(2)^3*z-T(2)^3-2*T(2)^2*T(5)*z+2*T(2)^2*T(5)+T(2)^2*T(6)+2*
norid[5]=T(2)^6-2*T(2)^4*T(5)-T(2)^4*T(6)+2*T(2)^4*z^2-T(2)^4*z+2*T(2)^4+T(2)^2*T(5)^2-T
norid[6]=T(2)^3*T(5)+2*T(2)^3*z+2*T(2)^2*T(5)-T(2)*T(5)^2-T(2)*T(5)*T(6)+T(2)^2*z-T(2)*T
norid[7]=T(2)^4*z+2*T(2)^4+T(2)^3*T(6)-T(2)^3*z^2+T(2)^3*z-2*T(2)^3-T(2)^2*T(5)*z-2*T(2)
norid[8]=T(2)^8-T(2)^7-2*T(2)^6*T(5)+2*T(2)^6+T(2)^5*T(5)+T(2)^4*T(5)^2+2*T(2)^5*T(6)+2*
norid[9]=2*T(2)^5+T(2)^4-2*T(2)^3*T(5)-2*T(2)^3-2*T(2)^2*T(5)+2*T(2)^2*T(6)+2*T(2)^2*z+2
norid[10]=T(2)^7-T(2)^6-2*T(2)^5*T(5)-T(2)^5+T(2)^4*T(5)+T(2)^3*T(5)^2+2*T(2)^4*T(6)+2*T
norid[11]=T(2)^6-T(2)^5-2*T(2)^4*T(5)+2*T(2)^4+T(2)^3*T(5)+T(2)^2*T(5)^2+2*T(2)^3*T(6)+2
norid[12]=2*T(2)^7+T(2)^5*T(5)+T(2)^5-2*T(2)^4*T(5)+2*T(2)^3*T(5)^2-T(2)^4*T(6)-T(2)^4*z
norid[13]=2*T(2)^6+T(2)^4*T(5)+2*T(2)^4*z+T(2)^4-2*T(2)^3*T(5)+2*T(2)^2*T(5)^2-T(2)^3*T(
norid[14]=T(2)^6+T(2)^5-2*T(2)^4*T(5)-T(2)^4*z+T(2)^4+2*T(2)^3*T(5)+T(2)^2*T(5)^2+2*T(2)
norid[15]=2*T(2)^4*T(5)-T(2)^4-T(2)^3*T(5)-2*T(2)^2*T(5)^2+2*T(2)^3-T(2)^2*T(5)+2*T(2)*T
norid[16]=2*T(2)^4*z+T(2)^4-T(2)^3*z-2*T(2)^2*T(5)*z-T(2)^2*T(5)-2*T(2)^2*z-T(2)^2-2*T(2
norid[17]=2*T(2)^4*z+T(2)^4+2*T(2)^3*T(5)-T(2)^3*z-2*T(2)^2*T(5)*z-2*T(2)^2*T(5)-2*T(2)*
norid[18]=T(2)^4*z-T(2)^4-T(2)^3*T(5)+2*T(2)^3*z+2*T(2)^3-T(2)^2*T(5)*z-2*T(2)^2*T(5)+T(
norid[19]=2*T(2)^4*z-2*T(2)^3*z+2*T(2)^3-2*T(2)^2*T(5)*z+2*T(2)^2*z-T(2)^2+T(2)*T(5)*z-2
norid[20]=2*T(2)^4*T(6)+T(2)^4*z+2*T(2)^4-T(2)^3*T(6)-2*T(2)^2*T(5)*T(6)+2*T(2)^3*z+T(2)
norid[21]=T(2)^4*z+T(2)^4-2*T(2)^3*T(6)+T(2)^3*z-2*T(2)^3-T(2)^2*T(5)*z-T(2)^2*T(5)+T(2)
norid[22]=2*T(2)^7-2*T(2)^6+T(2)^5*T(5)-2*T(2)^5*T(6)+2*T(2)^5*z-T(2)^4*T(5)+2*T(2)^3*T(
norid[23]=2*T(2)^7-2*T(2)^6+T(2)^5*T(5)-2*T(2)^5*T(6)+2*T(2)^5*z-T(2)^4*T(5)+2*T(2)^3*T(
norid[24]=2*T(2)^7+2*T(2)^6+T(2)^5*T(5)-2*T(2)^5*T(6)+2*T(2)^5*z+T(2)^5+T(2)^4*T(5)+2*T(
norid[25]=T(2)^21-T(2)^20-2*T(2)^19*T(5)-2*T(2)^19*T(6)+2*T(2)^19*z-2*T(2)^19-2*T(2)^18*
> option(redSB);
> ideal j=std(norid);j;
j[1]=T(5)^2*z^4+T(5)^2*z^3+T(5)^2*z^2-2*T(5)^2*z+2*T(2)*T(6)*z^4+2*T(2)*T(6)*z^3+T(2)*T(
j[2]=T(2)*T(5)-2*T(5)^2*z^3-2*T(5)^2*z^2-2*T(5)^2*z+T(2)*T(6)*z^3+T(2)*T(6)*z^2-2*T(2)*T
j[3]=T(2)^2-T(5)^2*z^3+2*T(5)^2*z^2+2*T(5)^2*z-2*T(5)^2-2*T(2)*T(6)*z^3-T(2)*T(6)*z^2+2*
j[4]=T(6)^3+T(5)^2*z^2-2*T(5)^2*z+T(5)^2+2*T(2)*T(6)*z^2-2*T(2)*T(6)*z+T(2)*T(6)-2*T(5)*
j[5]=T(5)*T(6)^2-T(5)^2*z^3+T(5)^2*z^2+T(5)^2-2*T(2)*T(6)*z^3+2*T(2)*T(6)*z^2-T(2)*T(6)*
```

```
j[6]=T(2)*T(6)^2+T(5)^2*z^2+2*T(5)^2+2*T(2)*T(6)*z^2-2*T(5)*T(6)*z^2+T(5)*T(6)*z+T(5)*T(
j[7]=T(5)^2*T(6)-2*T(5)^2*z^3-2*T(5)^2*z^2+2*T(5)^2*z+2*T(5)^2+T(2)*T(6)*z^3+T(2)*T(6)*z
j[8]=T(5)^3+T(5)^2*z^3-2*T(5)^2*z^2-2*T(5)^2*z-2*T(5)^2+2*T(2)*T(6)*z^3+T(2)*T(6)*z^2-T(
```