

In SINGULAR 3-1-0, unweighted,

```
> LIB "normal.lib";
> ring r3=3,(y,x),dp;
> ideal i3=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
> list nor3=normalP(i3,"withring");
> nor3;
[1]:[1]:
// characteristic : 3
// number of vars : 5
//      block 1 : ordering dp
//      : names T(1) T(2) T(4)
//      block 2 : ordering dp
//      : names y x
//      block 3 : ordering C
[2]:[1]:
  _[1]=y3x2-y2x2
  _[2]=y4x+y3x+y2x
  _[3]=y5+y4+y3
  _[4]=x8
  _[5]=yx4
[3]:[1]:16 [2]: 16
> def R3=nor3[1][1];
> setring R3;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=T(1)*y-T(1)-T(2)*x
norid[2]=T(1)*x^2-y^2+y
norid[3]=T(1)^2*y-T(1)^2-T(4)*x^3-y^2+y
norid[4]=T(2)^2-T(1)*y+T(1)-T(4)*y*x+T(4)*x
norid[5]=T(2)*T(4)-y^2*x-y*x-x
norid[6]=T(1)^2*T(4)-y^3-y^2-y
norid[7]=T(4)^2-T(2)*y+T(2)+T(4)*y^2-T(4)*y-y*x^4+x^4+y*x-x
norid[8]=T(1)^2*x-T(2)*y
norid[9]=T(4)*y-x^4
norid[10]=T(1)*T(2)-T(1)*x-T(4)*x^2
norid[11]=T(1)^3-T(1)*y-T(4)*y*x
norid[12]=T(1)^2*T(2)-T(2)*y+T(4)*y-y*x^4
norid[13]=T(1)^4-T(1)^2*y-y^2*x^3+y*x^3
norid[14]=T(1)*T(4)-y*x^2+x^2
norid[15]=x^11+y^3*x^4-y^6-y^2*x^4+y^3

> ring r5=5,(y,x),dp;
> ideal i5=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
```

```

> list nor5=normalP(i5,"withring");
> nor5;
[1]: [1]:
// characteristic : 5
// number of vars : 5
//      block 1 : ordering dp
//      : names T(1) T(2) T(3)
//      block 2 : ordering dp
//      : names y x
//      block 3 : ordering C
[2]: [1]:
  _[1]=y4x-2y3x+y2x
  _[2]=y5+2y4-2y3-y2
  _[3]=yx7-x7
  _[4]=x10
  _[5]=y2x3-yx3
[3]: [1]: 16 [2]: 16
> def R5=nor5[1][1];
> setring R5;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=T(1)*y-T(1)-T(2)*x
norid[2]=T(1)*x^2-y^2+y
norid[3]=T(1)^2*y-T(1)^2-T(3)*x^3-y^2+y
norid[4]=T(2)^2-T(1)*y+T(1)-T(3)*y*x+T(3)*x
norid[5]=T(2)*T(3)-y^2*x+2*y*x-x
norid[6]=T(3)^2-T(2)*y+T(2)+y*x-x
norid[7]=T(1)^2*T(3)-T(3)*y+x^4-y^3+2*y^2-y
norid[8]=T(1)^4-2*T(1)^2*y+T(1)^2+T(1)*x^2+T(3)*x^3-y^2*x^3+y*x^3
norid[9]=T(1)^2*x-T(2)*y
norid[10]=T(3)*y-x^4
norid[11]=T(1)*T(2)-T(1)*x-T(3)*x^2
norid[12]=T(1)*T(3)-y*x^2+x^2
norid[13]=T(1)^3-T(1)*y-T(3)*y*x
norid[14]=T(1)^2*T(2)-T(2)*y-y*x^4+x^4
norid[15]=x^11+y^3*x^4-y^6-y^2*x^4-2*y^5+2*y^4+y^3

> ring r7=7,(y,x),dp;
> ideal i7=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
> list nor7=normalP(i7,"withring");
> nor7;
[1]: [1]:
// characteristic : 7
// number of vars : 5

```

```

//      block 1 : ordering dp
//      : names  T(1) T(2) T(3)
//      block 2 : ordering dp
//      : names  y x
//      block 3 : ordering C
[2]: [1]:
  _[1]=y4x-2y3x+y2x
  _[2]=y5-3y4+3y3-y2
  _[3]=yx7-x7
  _[4]=x10
  _[5]=y2x3-yx3
[3]: [1]: 16 [2]: 16
> def R7=nor7[1][1];
> setring R7;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=T(1)*y-T(1)-T(2)*x
norid[2]=T(1)*x^2-y^2+y
norid[3]=T(1)^2*y-T(1)^2-T(3)*x^3-y^2+y
norid[4]=T(2)^2-T(1)*y+T(1)-T(3)*y*x+T(3)*x
norid[5]=T(2)*T(3)-y^2*x+2*y*x-x
norid[6]=T(3)^2-T(2)*y+T(2)+y*x-x
norid[7]=T(1)^2*T(3)-T(3)*y+x^4-y^3+2*y^2-y
norid[8]=T(1)^2*x-T(2)*y
norid[9]=T(3)*y-x^4
norid[10]=T(1)*T(2)-T(1)*x-T(3)*x^2
norid[11]=T(1)*T(3)-y*x^2+x^2
norid[12]=T(1)^3-T(1)*y-T(3)*y*x
norid[13]=T(1)^2*T(2)-T(2)*y-y*x^4+x^4
norid[14]=T(1)^4-T(1)^2*y-y^2*x^3+y*x^3
norid[15]=x^11+y^3*x^4-y^6-y^2*x^4+3*y^5-3*y^4+y^3

```

Note that the presentations are almost the same, though the fractions may not be. Compare this to char 0:

```

> ring r0=0,(y,x),dp;
> ideal i0=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
> list nor0=normal(i0);
> nor0;
[1]: [1]:
// characteristic : 0
// number of vars : 7
//      block 1 : ordering dp
//      : names  T(1) T(2) T(3) T(4) T(5)
//      block 2 : ordering dp

```

```

//          : names    y x
//      block 3 : ordering C
[2]: [1]:
  _[1]=3yx10+3x10+3y4x3-2y3x3-y2x3
  _[2]=3y4x4-2x8-3y3x4-3y2x4+3yx4
  _[3]=3yx9+3x9+3y4x2-2y3x2-y2x2
  _[4]=3y2x8-3x8+2yx5-2y4x+4y3x-2y2x
  _[5]=9y3x7-9yx7+6y2x4-4yx4+6y4-12y3+6y2
  _[6]=3y5-2yx4-3y4-3y3+3y2
> def R0=nor0[1][1];
> setring R0;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=-T(1)+T(3)*x
norid[2]=3*T(4)*y-T(5)*x+2*x
norid[3]=-T(3)*y+T(3)+T(4)*x+x^2
norid[4]=-T(1)*x+y^2-y
norid[5]=3*T(2)*x^3-T(5)*y+T(5)+2*y-2
norid[6]=-T(1)*y*x+T(1)*x+T(2)*y+T(4)*x^3
norid[7]=-9*T(1)*x^3+9*T(3)*y^3-12*T(3)*y^2+T(3)*y+2*T(3)
          +2*T(4)*x-3*T(5)*y*x^2+T(5)*x^2
norid[8]=-3*T(1)*y^2*x+3*T(1)*y*x+3*T(2)*y^2-2*T(2)*y+T(5)*x^4
norid[9]=9*T(1)*x^4-9*T(1)*y^3+12*T(1)*y^2-T(1)*y-2*T(1)
          -2*T(4)*x^2+3*T(5)*y*x^3-T(5)*x^3
norid[10]=T(1)*x^5-T(2)*y^3+T(2)*y^2
norid[11]=T(1)^2-T(3)*y^2+T(3)+T(4)*x+x^2
norid[12]=T(1)*T(2)-y*x^3+x^3
norid[13]=T(2)^2-T(4)*y+T(4)
norid[14]=T(1)*T(3)-T(4)*y-y*x
norid[15]=T(2)*T(3)-y*x^2+x^2
norid[16]=T(3)^2-1/3*T(5)-y+2/3
norid[17]=T(1)*T(4)-T(2)*x^3
norid[18]=T(2)*T(4)+T(2)*x-y^2*x+2*y*x-x
norid[19]=T(3)*T(4)-T(2)*x^2
norid[20]=T(4)^2+T(2)*x+T(3)*y-T(3)-x^5-x^2
norid[21]=T(1)*T(5)-2*T(3)*x-3*x^6
norid[22]=T(2)*T(5)-2*T(2)+3*x^4-3*y^3+6*y^2-3*y
norid[23]=T(3)*T(5)-2*T(3)-3*x^5
norid[24]=T(4)*T(5)+3*T(2)*y+3*T(4)*y-2*T(4)-3*y*x^4
norid[25]=T(5)^2+9*T(2)*x^3-T(5)-9*y^2*x^3+9*y*x^3-2
norid[26]=x^11+y^3*x^4-y^6-y^2*x^4+3*y^5-3*y^4+y^3

```

This should have been the same for this example, but doesn't even have the same number of variables.

In positive characteristic the new weights are (10,15,13) with 20 suppressed.
 In char 0, the new weights are (16,13,10,15,20) with $T(1) = T(3)x$ superfluous.
 This is easily checked by the inspired code:

```
> intmat B[6][6]=
  1, 1, 1, 1, 1, 0,
  1, 1, 1, 1, 0, 0,
  1, 1, 1, 0, 0, 0,
  1, 1, 0, 0, 0, 0,
  1, 0, 0, 0, 0, 0,
20,15,13,11,10,6;
> ring ic0=0, (f20,f15,f13,f11,f10,f6),M(B);
> map phi=R0,f10*f6,f13,f10,f15,f20,f11,f6;
> ideal j0=std(phi(norid));j0;
j0[1]=3*f10^2-f20-3*f11+2
j0[2]=f11*f10-f15*f6-f10-f6^2
j0[3]=f13*f10-f11*f6^2+f6^2
j0[4]=f15*f10-f13*f6^2
j0[5]=f20*f10-2*f10-3*f6^5
j0[6]=f11^2-f11-f10*f6^2
j0[7]=f13*f11-f6^4
j0[8]=3*f15*f11-f20*f6+2*f6
j0[9]=f20*f11-f20-3*f13*f6^3-2*f11+2
j0[10]=3*f13^2-f20*f6+3*f15+2*f6
j0[11]=f15*f13+f13*f6+f11*f6-f10*f6^3-f6
j0[12]=f20*f13-3*f15*f6^3-2*f13
j0[13]=f15^2+f15*f6+f13*f6-f6^5
j0[14]=f20*f15+f20*f6-2*f15-3*f11*f6^4+3*f6^4-2*f6
j0[15]=f20^2-f20+9*f13*f6^3-9*f10*f6^5-2
```

Macaulay2 version 1.2

```
i10 : R=ZZ/3[y,x]
i11 : I=ideal((y^2-y)^3-y*x^4*(y^2-y)-x^11)
i12 : S=R/I
i13 : F=icFracP(S)

o13 = {1,  $\frac{y^2 - y}{x^2}$ ,  $\frac{y^3 + y^2 + y}{x^3}$ ,  $\frac{y^4 + y^3 + y^2}{x^4}$ ,  $\frac{x}{y}}$ }
```

```
i14 : P=presentation(integralClosure(S))
```

```
o14 = | w_4x2-y2+y
      w_4^3-w_4y-x5
      w_7x+w_4y-w_4
      w_7y+w_4^2x
      w_9x+w_7^2
      w_9y+w_4^2x+yx4-x4
      w_9w_4-w_7w_4+y2x2+yx2+x2
      w_9w_7-w_7^2-y3x+x
      w_9^2-w_7^2-w_7-w_4^2yx-x8+y3x+y2x+x |
```

```
o14 : Matrix (ZZ, ZZ, ZZ, ZZ, ZZ, ZZ, ZZ, ZZ, ZZ, ZZ)
      1 1 1 1 1 1 1 1 1 1
      3 9 7 4 3 9 7 4 3 9 7 4
```

```
i1 : R=ZZ/3[y,x];
i2 : I=ideal((y^2-y)^3-y*x^4*(y^2-y)-x^11);
i3 : S=R/I;
i4 : time F=icFracP(S)
      -- used 0.17 seconds
```

```
o4 = {1,  $\frac{y^2 - y}{x^2}$ ,  $\frac{y^3 + y^2 + y}{x^3}$ ,  $\frac{y^4 + y^3 + y^2}{x^4}$ ,  $\frac{x}{y}}$ }
```

```
i5 : time P=presentation(integralClosure(S))
      -- used 0.53 seconds
```

```
o5 = | w_(3,0)x2-y2+y
      w_(3,0)^3-w_(3,0)y-x5
```

$w_-(5,0)x-w_-(3,0)y+w_-(3,0)$
 $w_-(5,0)y-w_-(3,0)^2x$
 $w_-(6,0)x-w_-(5,0)^2$
 $w_-(6,0)y-w_-(3,0)^2x-yx^4+x^4$
 $w_-(6,0)w_-(3,0)-w_-(5,0)w_-(3,0)-y^2x^2-yx^2-x^2$
 $w_-(6,0)w_-(5,0)-w_-(5,0)^2-y^3x+x$
 $w_-(6,0)^2-w_-(5,0)^2+w_-(5,0)-w_-(3,0)^2yx-x^8+y^3x+y^2x+x \mid$

The fractions have implicit weights (0,10,15,20,13), while the new variables in the presentation have implicit weights (24,15,10). Clearly (?) either $w_{6,0} - x^4$ or $w_{6,0} + x^4$ has implicit weight 13, so while this seems to be technically correct,...

```

i6 : R=ZZ/5[y,x];
i7 : I=ideal((y^2-y)^3-y*x^4*(y^2-y)-x^11);
i8 : S=R/I;
i9 : time F=icFracP(S)
-- used 2.87 seconds

```

```

o9 = {1, -----,
          6      5      4      4      3      4      3      2
          y  - 2y  + y*x  - y  + 2y x  - 2x  + y  + y  - 2y*x
          -----,
          4      3      2
          x  - 2y  + 2y

```

```

-----
- y x - 2y*x  - 2y x + x  - y x - 2y x  + 2y x + 2y*x  + 2y*x
-----,
          4      3      2
          x  - 2y  - y  - 2y

```

```

-----
x  + 2y  - 2y  + y  y x  - y x  - y  + y
-----, -----}
          y          4      3      2
          x  - 2y  + 2y

```

```

i30 : time P=presentation(integralClosure(S))
-- used 0.53 seconds

```

```

o30 = | w_(3,0)x2-y2+y
      w_(3,0)^3-w_(3,0)y-x5
      w_(5,0)x-w_(3,0)y+w_(3,0)
      w_(5,0)y-w_(3,0)^2x
      w_(6,0)x-w_(5,0)^2
      w_(6,0)y-w_(3,0)^2x-yx4+x4
      w_(6,0)w_(3,0)-w_(5,0)w_(3,0)-y2x2+2yx2-x2
      w_(6,0)w_(5,0)-w_(5,0)^2-y3x-2y2x+2yx+x
      w_(6,0)^2-w_(5,0)^2+w_(5,0)+2w_(3,0)^2yx+2w_(3,0)^2x-x8-2y3x-y2x+2yx+x |

```

while also probably technically correct, gives fractions that are in desperate need of reduction modulo the original ring. As one does this one notices that the denominators may change from one reduction to the next, since Macaulay2 has a strange idea (actually I believe no idea, though it is based on certain syzygy computations) of what a canonical form of a rational function should be.

If one does reduce them, then one probably gets functions with implicit weights (0,15,19,13,20), with the function of weight 10 recoverable as a combination of those of weights 19 and 13.

At least the old errors encountered by *presentation(integralClosure(S))* are no longer there.

MAGMA easily gives:

```

1 1
2 f11
3 1/f6^2*f11^2+22/f6^2*f11
4 1/f6^3*f11^3+21/f6^3*f11^2+1/f6^3*f11
5 1/f6^4*f11^4+21/f6^4*f11^3+1/f6^4*f11^2
6 1/f6^7*f11^5+20/f6^7*f11^4+3/f6^7*f11^3+(22*f6^4+22)/f6^7*f11^2+1/f6^3*f11

```

in characteristic 23, and

```

1 1
2 f11
3 1/f6^2*f11^2-1/f6^2*f11
4 1/f6^3*f11^3-2/f6^3*f11^2+1/f6^3*f11
5 1/f6^4*f11^4-2/f6^4*f11^3+1/f6^4*f11^2
6 1/f6^7*f11^5-3/f6^7*f11^4+3/f6^7*f11^3+(-f6^4-1)/f6^7*f11^2+1/f6^3*f11

```

over the rationals,
but as always, no “stand-alone” presentations.
MAGMA’s Normalisation function

```
t:=Cputime();
F:=Rationals();
P<y,x>:=PolynomialRing(F,2,"weight",[1,0,11,6]);
f:=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
I:=ideal<P|f>;
N:=Normalisation(I);
J:=N[1][1];J;
"Normalisation time=",Cputime(t);

Basis:
[
  -J.1^2*J.2 + J.3^2 - J.3,
  -J.1*J.5 + J.2*J.3 - J.2,
  -J.1*J.2^2 + J.3*J.5,
  -J.1^3*J.4 - J.1^2*J.2 + J.2^2*J.3 - J.2^2,
  -J.1^4 + J.3*J.4,
  -J.1^2*J.4 - J.1*J.2 + J.2*J.5,
  -J.1^5 - J.1*J.5 + J.2^3 - J.2,
  -J.1^2*J.3 + J.1^2 + J.2*J.4,
  -J.1^5 + J.1*J.4 - J.1*J.5 + J.5^2,
  -J.1^4*J.3 + J.1^4 - J.1*J.2^2 + J.2^2*J.5,
  -J.1^3*J.2 + J.1*J.3 - J.1 + J.4*J.5,
  -J.1^5*J.2 - J.1^3*J.4 - J.1^2*J.2 + J.2^4 - J.2^2,
  -J.1^3*J.5 + J.2^2*J.4,
  -J.1*J.2^2 + J.1*J.3 - J.1 + J.4^2 + J.5
]
Normalisation time= 0.380
```

but the output of

```
G:=GroebnerBasis(J);G;
"total time=",Cputime(t);
```

is about 4000 line long, so omitted. An inspired choice of ordering

```
t:=Cputime();
F:=Rationals();
P<e,d,c,b,a>:=PolynomialRing(F,5,"grevlex",[15,13,11,10,6]);

f1:=-a^2*b + c^2 - c;
f2:=-a*e + b*c - b;
f3:=-a*b^2 + c*e;
f4:=-a^3*d - a^2*b + b^2*c - b^2;
f5:=-a^4 + c*d;
```

```

f6:=-a^2*d - a*b + b*e;
f7:=-a^5 - a*e + b^3 - b;
f8:=-a^2*c + a^2 + b*d;
f9:=-a^5 + a*d - a*e + e^2;
f10:=-a^4*c + a^4 - a*b^2 + b^2*e;
f11:=-a^3*b + a*c - a + d*e;
f12:=-a^5*b - a^3*d - a^2*b + b^4 - b^2;
f13:=-a^3*e + b^2*d;
f14:=-a*b^2 + a*c - a + d^2 + e;

I:=ideal<P|f1,f2,f3,f4,f5,f6,f7,f8,f9,f10,f11,f12,f13,f14>;
G:=GroebnerBasis(I);G;
Cputime(t);

```

gives an acceptable answer:

```

e^2 - a^5 - e*a + d*a,
b^3 - a^5 - e*a - b,
e*d - b*a^3 + c*a - a,
d^2 - b^2*a + c*a + e - a,
e*c - b^2*a,
e*b - d*a^2 - b*a,
d*c - a^4,
d*b - c*a^2 + a^2,
c^2 - b*a^2 - c,
c*b - e*a - b

```

0.010

though again missing b^2 of weight 20 from my algebra perspective.

normalP in SINGULAR 3-1-0 and the newer “withRing” and “noRed” options produced:

```

                                SINGULAR                               /
A Computer Algebra System for Polynomial Computations / version 3-1-0
                                0<
                                by: G.-M. Greuel, G. Pfister, H. Schoenemann \ Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern \
> LIB "/home/leonada/presolve.lib";
> LIB "/home/leonada/normal.lib";
> ring r=3,(y,x),wp(11,6);
> ideal i=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
> list norp=normalP(i,"withRing","noRed");
> norp;
[1]:
    [1]:

```

```

// characteristic : 3
// number of vars : 6
//      block 1 : ordering dp
//      : names  T(1) T(2) T(3) T(4)
//      block 2 : ordering wp
//      : names  y x
//      : weights 11 6
//      block 3 : ordering C
[2]:
  [1]:
    _[1]=y3x2-y2x2
    _[2]=x8
    _[3]=y4x+y3x+y2x
    _[4]=y5+y4+y3
    _[5]=yx4
[3]:
  [1]:
    16
  [2]:
    16
> def R=norfp[1][1];
> setring R;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=T(1)*y-T(1)-T(3)*x
norid[2]=T(3)*y-T(4)*x
norid[3]=T(1)*x^2+T(2)*x^3-T(4)*y+T(4)
norid[4]=-T(1)*x^2+y^2-y
norid[5]=-T(2)*y+x^4
norid[6]=T(1)^2-T(4)
norid[7]=T(1)*T(2)-y*x^2+x^2
norid[8]=T(2)^2+T(3)-T(4)*x+y*x-x
norid[9]=T(1)*T(3)-T(1)*x-T(2)*x^2
norid[10]=T(2)*T(3)-y^2*x-y*x-x
norid[11]=T(3)^2-T(1)*y+T(1)+T(2)*x-x^5
norid[12]=T(1)*T(4)-T(1)*y-x^5
norid[13]=T(2)*T(4)-y^3-y^2-y
norid[14]=T(3)*T(4)-T(4)*x-y*x^4+x^4
norid[15]=T(4)^2-T(2)*x^3-T(4)-y^2*x^3+y*x^3-y^2+y
norid[16]=y^6-x^11-y^3*x^4+y^2*x^4-y^3
> option(redSB);
> ideal j=std(norid);j;
j[1]=y^6-x^11-y^3*x^4+y^2*x^4-y^3
j[2]=T(4)*y^2-T(4)*y-x^7-y^3+y^2

```

```

j[3]=T(4)*x^4-y^4-y^3-y^2
j[4]=T(3)*y-T(4)*x
j[5]=T(3)*x^3-y^3-y^2-y
j[6]=T(2)*y-x^4
j[7]=T(2)*x^3-T(4)*y+T(4)+y^2-y
j[8]=T(1)*y-T(1)-T(3)*x
j[9]=T(1)*x^2-y^2+y
j[10]=T(4)^2-T(4)*y-y^2*x^3+y*x^3
j[11]=T(3)*T(4)-T(4)*x-y*x^4+x^4
j[12]=T(2)*T(4)-y^3-y^2-y
j[13]=T(1)*T(4)-T(1)-T(3)*x-x^5
j[14]=T(3)^2+T(2)*x-T(3)*x-x^5
j[15]=T(2)*T(3)-y^2*x-y*x-x
j[16]=T(1)*T(3)-T(1)*x-T(2)*x^2
j[17]=T(2)^2+T(3)-T(4)*x+y*x-x
j[18]=T(1)*T(2)-y*x^2+x^2
j[19]=T(1)^2-T(4)

```

```

> ring r=5,(y,x),wp(11,6);
> ideal i=(y^2-y)^3-y*x^4*(y^2-y)-x^11;
> list norp=normalP(i,"withRing","noRed");
> norp;
[1]:
  [1]:
// characteristic : 5
// number of vars : 6
//      block 1 : ordering dp
//                : names  T(1) T(2) T(3) T(4)
//      block 2 : ordering wp
//                : names  y x
//                : weights 11 6
//      block 3 : ordering C
[2]:
  [1]:
  _[1]=x11+y3x4+y5-y2x4-2y4+y3
  _[2]=y3x6-y2x6
  _[3]=yx10
  _[4]=y2x9
  _[5]=y4x2-y3x2
[3]:
  [1]:
  16
  [2]:
  16
> def R=norp[1][1];

```

```

// ** redefining R **
> setring R;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=T(3)*y-T(4)*x
norid[2]=-T(1)*y+T(1)+T(3)*x+x^2
norid[3]=T(2)*x^3-T(4)*y+T(4)
norid[4]=-T(1)*x^2+y^2-y
norid[5]=T(1)*y*x^2-T(1)*x^2-T(2)*y-T(3)*x^3
norid[6]=T(1)^2-T(4)-y
norid[7]=T(1)*T(2)-y*x^2+x^2
norid[8]=T(2)^2+T(3)-T(4)*x
norid[9]=T(1)*T(3)-T(2)*x^2
norid[10]=T(2)*T(3)+T(2)*x-y^2*x+2*y*x-x
norid[11]=T(3)^2+T(2)*x+T(3)*x-x^5
norid[12]=T(1)*T(4)-x^5
norid[13]=T(2)*T(4)-T(3)*x^3
norid[14]=T(3)*T(4)+T(3)*y-y*x^4+x^4
norid[15]=T(4)^2+T(2)*x^3+T(4)-y^2*x^3+y*x^3
norid[16]=y^6-x^11-y^3*x^4+2*y^5+y^2*x^4-2*y^4-y^3
> option(redSB);
> ideal j=std(norid);j;
j[1]=y^6-x^11-y^3*x^4+2*y^5+y^2*x^4-2*y^4-y^3
j[2]=T(4)*y^2-T(4)*y-x^7
j[3]=T(4)*x^4-y^4+y*x^4+2*y^3-y^2
j[4]=T(3)*y-T(4)*x
j[5]=T(3)*x^3-y^3+x^4+2*y^2-y
j[6]=T(2)*y-x^4
j[7]=T(2)*x^3-T(4)*y+T(4)
j[8]=T(1)*y-T(1)-T(3)*x-x^2
j[9]=T(1)*x^2-y^2+y
j[10]=T(4)^2+T(4)*y-y^2*x^3+y*x^3
j[11]=T(3)*T(4)+T(4)*x-y*x^4+x^4
j[12]=T(2)*T(4)-y^3+x^4+2*y^2-y
j[13]=T(1)*T(4)-x^5
j[14]=T(3)^2+T(2)*x+T(3)*x-x^5
j[15]=T(2)*T(3)+T(2)*x-y^2*x+2*y*x-x
j[16]=T(1)*T(3)-T(2)*x^2
j[17]=T(2)^2+T(3)-T(4)*x
j[18]=T(1)*T(2)-y*x^2+x^2
j[19]=T(1)^2-T(4)-y

```

with different fractions but of weights 10,13,15,20, as expected.

```
> ring r=7,(y,x),wp(11,6);
```

```
> ideal i=(y^2-y)^3-y*x^4*(y^2-y)-x^11;  
> list norp=normalP(i,"withRing","noRed");
```

seems to be too hard for it.